12.1 Introduction

The demand for automobiles with less fuel consumption and less emissions has increased since the year 2000 because of rising commodity prices. However, the quest for lightweight designs is (partly) counterbalanced by the additional weight related to passive and active safety systems and the steadily increasing comfort level for passengers. This creates a demand for intelligent lightweight concepts in many industries, not only in aerospace and automotive.

Besides the lightweight design, the increasing usage of optimization software leads to thin-walled and slender components which tend to buckle under axial loading.

12.2 Buckling

Thin structures subject to compression loads that haven’t achieved the material strength limits can show a failure mode called buckling. Buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding (http://en.wikipedia.org/wiki/Buckling).

In other words, once a critical load is reached, the slender component (for example a beam where its length is much larger than its cross sectional area) draws aside instead of taking up additional load.
This failure can be analyzed using a technique well known as **linear buckling analysis**. The goal of this analysis is to determine the buckling load factor, $\lambda$, and the critical buckling load.

In RADIOSS, if the load factor $\lambda$ is > 1, the component is considered to be safe (i.e. the actual load can be multiplied by $\lambda$ until buckling would occur).

### 12.3 Elastic Buckling

In 1757 Leonhard Euler derived the following equation:

$$ F_{\text{crit}} = \frac{\pi^2 E I}{(K L)^2} = \frac{\pi^2 E I}{5^2} $$

where

- $F$ = maximum or critical force (vertical load on column)
- $E$ = modulus of elasticity
- $I$ = area moment of inertia (second moment of area)
- $L$ = unsupported length of column
- $K$ = column effective length factor, whose value depends on the conditions of the end support of the column, as follows:
For both ends pinned (hinged, free to rotate), $K = 1.0$

For both ends fixed, $K = 0.50$

For one end fixed and the other end pinned, $K = 0.707$

For one end fixed and the other end free to move laterally, $K = 2.0$

$KL$ = the effective buckling length of the column

In other words, the critical force depends on:

- Length of column
- Cross-section (second moment of surface area)
- Material property (Young's modulus, in case of elastic material)
- Boundary condition (The boundary conditions determine the mode of bending and the distance between inflection points on the deflected column. The closer together the inflection points are, the higher the resulting capacity of the column.)

The strength of a column may therefore be increased by distributing the material so as to increase the moment of inertia. This can be done without increasing the weight of the column by distributing the material as far from the principal axis of the cross section as possible, while keeping the material thick enough to prevent local buckling. This bears out the well-known fact that a tubular section is much more efficient than a solid section for column service.

Note: Real constructions often contain imperfections, such as pre-deformations, due to which large displacements or failure may occur even if the loading is still below the ideally critical load. The linear buckling analysis in general overestimates the strength/stability of the structure and leads to non-conservative results. Thus, it shouldn't be used as the only measure. However, the linear buckling analysis at least provides information about the expected deformation shapes.

Looking at Euler's equation and dividing it by the area $A$ defines the critical stress at which buckling will occur:

$$\sigma_{krit} = \frac{F_{krit}}{A} \leq R_e$$

where $R_e$ is the elastic limit.

$$\sigma_{krit} = \frac{F_{krit}}{A} = \frac{\pi^2 E I}{A s^2} = \frac{\pi^2 E}{\lambda^2} \leq R_e$$

with

$$\lambda = \frac{s}{\sqrt{(l/A)}}$$
12.4 Linear Stability Analysis with RADIOSS

The problem of linear buckling in finite element analysis is solved by first applying a reference level of loading, $F_{ref}$, to the structure. This is ideally a unit load, $F$, that is applied. The unit load and respective constraints, SPC, are referenced in the first load steps/subcase. A standard linear static analysis is then carried out to obtain stresses which are needed to form the geometric stiffness matrix $K_u$. The buckling loads are then calculated as part of the second load steps/subcase, by solving an eigenvalue problem:

$$(K - \lambda K_u)\mathbf{x} = 0$$

$K$ is the stiffness matrix of the structure and $\lambda$ is the multiplier to the reference load. The solution of the eigenvalue problem generally yields $n$ eigenvalues $\lambda_i$ (buckling load factor) where $n$ is the number of degrees of freedom (in practice, only a subset of eigenvalues is usually calculated). The vector $\mathbf{x}$ is the eigenvector corresponding to the eigenvalue.

The eigenvalue problem is solved using a matrix method called the Lanczos method. Not all eigenvalues are required. Only a small number of the lowest eigenvalues are normally calculated for buckling analysis.

The lowest eigenvalue is associated with buckling. The critical or buckling load is:

$$F_{crit} = \lambda_{crit} F_{ref}$$

In other words,

$$\lambda_{crit} = F_{crit} / F_{ref}$$

thus

$$\lambda < 1 \text{ buckling}$$

$$\lambda > 1 \text{ safe}$$

Note: The displacement results obtained with a buckling analysis depict the buckling mode shape. Any displacement values are meaningless. The same holds true for stress and strain results from a buckling analysis.

From Theory to Practice: How to Set Up a Linear Buckling Analysis

In order to run a linear buckling analysis, the following two steps are required:

Step 1

3 load collectors and 2 load steps/subcases must be specified:

- One Load collector for constraints (SPC = SinglePoint Constraints; no Card Image needed)
- One Load collector for loads (ideally unit load); no Card Image needed
- One load collector (with Card Image ElGRL) which defines the number of buckling modes to be determined (card image shown below)
• ND = 1: This tells RADIOSS to extract the first buckling mode

Step 2

Defining 2 loadsteps; one for static and for buckling. The load collectors with the SPCs and the unit load define a static load subcase. Notice that the type is set to linear static:

Then the buckling load step (which needs the results from the static loadstep) is defined:

As before SPC references the load collector which contains the model constraints. STATSUB (BUCKLING) is referencing the static subcase/loadstep from before, and finally, METHOD references the load collector which bears the information about the number of eigen (buckling) modes to be extracted (i.e. load collector with Card Image EIGRL). Also notice that the type is now set to linear buckling.

Note:

• STATSUB cannot refer to a subcase that uses inertia relief.

• The buckling analysis will ignore zero-dimensional elements, MPC, RBE3, and CBUSH elements. These elements can be used in buckling analysis, but they do not contribute to the geometric stiffness matrix, Kc.

• By default, the contribution from the rigid elements to the geometric stiffness matrix is not included. Users have to add PARAM, KGRGD, YES to the bulk data section to include the contribution of rigid elements to the geometric stiffness matrix.

• In addition, through the EXCLUDE subcase information entry, users may decide to omit the contribution of other elements to the geometric stiffness matrix, effectively allowing users to control which parts of the structure are analyzed for buckling. The excluded properties are only removed from the geometric stiffness matrix, resulting in a buckling analysis with elastic boundary conditions. This means that the excluded properties may still be showing movement in the buckling mode.