Abstract

The research, described in this paper, focuses on the application of structural topology optimisation to the design of steel I-section beam web openings. A topology optimisation study is performed on the web of a steel I-section beam. A beam web design is then proposed based on the results of the topology optimisation study. A nonlinear finite element analysis technique is employed to determine the load carrying performance of the optimised beam in comparison with the conventional, widely used, cellular type beam. It is found that the optimised beam performs better in terms of load carrying capacities and stress intensities. Barriers to the implementation of the topology optimisation technique to the routine design of beam web are discussed.

A parametric topology optimisation study is conducted in order to determine the optimum opening topology for the wide range of beam cross sections that are found in practice. Thereafter, a generalised optimum web opening configuration is suggested based on the results of the parametric study. It is suggested that a shape optimisation study should be a next step towards maximising the efficiency of beams with this optimum web opening configuration.

Keywords: castellated beam, cellular beam, web opening, finite element analysis, topology optimisation.

1 Introduction

The research focuses on steel I-section beams with perforations in the web which are referred to as castellated beams, cellular beams and beams with web openings. The most important benefits when using such sections are the improved weight to stiffness ratio, ability to integrate building services into the structural depth and the perceived aesthetic appeal of the section [1]. The literature on the structural
behaviour of steel I-section beams with standard circular, rectangular and hexagonal web-opening configurations is extensive [2-6]. The constant desire for improvement and mature level of understanding of the structural action of perforated steel sections has recently led to novel opening shapes being investigated [7,8]. Novel opening shapes are claimed to have beneficial fabrication, structural performance, aesthetic qualities and improved structural efficiency when compared to standard opening types.

All of the opening shapes and configurations previously considered in the literature, are constrained by the requirement that they can be manufactured by the profile cutting procedure. The profile cutting procedure, also known as castellation, is a method of manufacturing beams with web openings whereby a “parent” steel I-section is selected and a zig-zag pattern is cut along the web of the section. The sections is then expanded and welded along the teeth of the zig-zag to form a deeper section with web openings (Figure 1a). It is obvious that if a beam is to be formed using this process the possible geometry of the web openings is constrained. The profile cutting procedure is currently regarded as the most economical and standardised method of fabricating beams with web openings. The only exception being when wide elongated web openings are considered.

More recently an alternative fabrication technique, plate assembly (Figure 1b), has been adopted by several leading fabricators of cellular beams. The plate assembly technique offers significantly increased design freedom in terms of the shape and layout of web openings that can be realised. It is, however, considered an expensive solution due the long welding line along the plates and the demanding requirements for accuracy when the perforations are engraved.

To this point, it could not be found in the literature that any formal structural optimisation studies had been performed with the aim of improving the design of steel I-section beam web opening shapes. Previous advancements in web opening configurations have rather relied upon engineering intuition and experience.

Figure 1 - a) Castellation Manufacturing Process, b) Plate Assembly Manufacturing Process
2 Scope of Work and Objectives

The scope of the present research is to investigate enhancements to the performance of steel I-section beams with web openings using formalised structural optimisation techniques. Specifically, structural topology optimisation is to be applied to the design of the beam web for the first time, investigating the alternatives for perforated beams which fulfil certain boundary criteria.

The objectives of this study are:

- to apply a structural topology optimisation technique to the design of a steel I-section beam web,
- to investigate the structural behaviour of a simply supported beam subjected to vertical shear actions with a topology optimised web, using the nonlinear finite (FE) element analysis, and
- To suggest an effective opening configuration for a wide variety of beam cross sections found in practice through an extensive parametric study

3 Initial Study on a Full Length Beam Section

Initially, a topology optimisation study was performed on the web of a simply supported steel I-section beam of a 5m span. A standard UB 305x165x40 was selected on the basis that it is a fairly common section to find in practice and mainly in building applications. The topology optimisation technique was used. Thereafter, the structural behaviour of the optimised beam was compared to a similar beam but with circular web openings, by carrying out a nonlinear FE analysis. The topology optimisation studies were performed using Altair Engineering’s Optistruct software. The comparative nonlinear FE analysis studies were performed using ANSYS.

3.1 Topology Optimisation Approach

Structural topology optimisation is concerned with the identification of the optimum number and location of openings, within a defined designable structural continuum to fulfil a given objective, subject to applied loading and constraints. Numerous techniques for the solution of structural topology optimisation problems have been suggested [9-12]. The solid isotropic material with penalisation (SIMP) technique [13] is currently cited as being the most prevalent method of solving topology optimisation problems with the firmest mathematical basis [14,15].

A FE model of the beam section was created to determine the structural response of the beam for use in the optimisation algorithm. The beam was modelled using shell elements with a nominal size of 10 mm. A linear elastic material model was used with Young’s modulus and Poisson’s ratio of 200 GPa and 0.3, respectively.
Uniform pressure loading was applied to the top flange of the beam. Constraints were applied to the end of the lower flange to model the support conditions. A standard linear static FE analysis approach was employed.

The maximisation of structural stiffness subject to a constraint on the available material has been shown to be effective when attempting to identify conceptual designs [16]. The optimisation approach taken was, therefore, one of minimising compliance subject to a constraint on the available material.

In order to prevent the emergence of small unmanufacturable scale features in the topology optimisation results so called minimum member size control was used. A minimum member size of 30mm was specified in order to prevent any design features at a scale smaller than this from emerging in the topology optimisation results. It is worth noting that only the web of the beam was defined as designable in the topology optimisation studies.

As it was aforementioned, the SIMP technique was used to solve the topology optimisation problem. In order to identify a discrete solid-void final design using the SIMP technique, it is necessary to specify an appropriate value for the intermediate density material penalisation factor [17]. The most appropriate penalisation factor for this specific problem was investigated and it was found that a penalisation factor of four was satisfactory.

The topology optimisation study was performed initially considering variations in the maximum available material constraint. The material constraint is specified in terms of a volume fraction, where the maximum material allowed in the final design is described as a fraction of the material present at the beginning of the optimisation. Volume fraction constraints of 0.5, 0.4 and 0.3 were studied.

### 3.2 Results

The results of the topology optimisation studies are presented in the form of element density plots in Figure 2. Note that the element density plots represent the optimal material distribution upon convergence of the optimisation. The red (light colour) zones represent the solid material, while the blue (dark colour) zones represent the suggested locations for openings. The transitional zones represent material of an intermediate density.

It is observed that a complex and irregular truss type design was formed. It is also noted that more material appeared to be distributed to areas of high shear, towards the supports. Where bending forces are predominant, towards the mid-span, the topology optimisation suggested the placement of no web material. It is also appeared that the beam web elements, as suggested by the topology optimisation, follow lines of principle stress in the beam web.
Following, a symmetry constrained topology optimisation was performed. Lines of symmetry about the centreline and longitudinal axis of the beam were specified and a topology optimisation was performed using a volume fraction constraint of 0.4. The results of the symmetry constrained topology optimisation study (Figure 3) showed a more rational truss type design with rhomboidal web openings which periodically change along the length of the beam. Similarly, a large opening was suggested at the mid-span of the beam. It became visible that the material is distributed within the beam web according to the ratio of shear and bending moment acting on the section.

It is evident that none of the designs suggested in the topology optimisation study could be manufacturable using the profile cutting procedure. The plate assembly technique would, therefore, need to be implemented to realise the designs.
3.3 Comparison of Cellular and Topology Optimised Beam

3.3.1 Post-Processing Topology Optimisation Results

It was necessary to post-process the topology optimisation results in order to define the beam web geometry to be used in the comparative study. The first step in the post processing was completed using the OSSmooth feature of Altair Engineering’s HyperMesh FE pre- and post-processing software [18]. An element density threshold was specified defining the element density above which the design was interpreted as solid geometric surfaces. Thereafter, Laplacian smoothing is performed by OSSmooth to smooth the boundaries of the design. Following the initial geometry extraction using OSSmooth, a manual intervention was necessary in order to define the geometry of the beam web as a series of curves in AutoCAD.

3.3.2 FE Model

Cellular beams often exhibit complex structural behaviour including localised buckling modes or yielding and redistribution of stresses around openings. A FE technique has been previously presented [19] that can capture these complex failure modes. The basis of the FE analysis technique employed is a three step process whereby an initial pre-stress was applied to the model and a linear static analysis was performed. The results of the linear static analysis were then used in an eigenvalue analysis to determine the first buckling frequency and the associated mode shape. Imperfections were applied to the FE mesh, using the first mode shape with a magnitude of the web thickness divided by 200. A geometric and materially nonlinear FE analysis is then performed to determine the load at which the beam buckles. The analysis was performed using the FE solver of ANSYS. A full detailed description of the technique, along with a verification of the results against experimental data, can be found in [20, 21].

The Young’s modulus, Poisson’s ratio, yield stress and tangent modulus were defined as 200 GPA, 0.3, 355 MPa and 2000 MPa, respectively, using a bi-linear elasto-plastic material model.
3.3.4 FE Study

The topology optimised beam web design was compared with a cellular type beam with an identical UB 305x165x40 cross section. To compare the relative efficiency of the beams, in terms of material usage versus load capacity, the cellular beam was specified such that its mass is as similar as possible to the mass of the optimised beam (Figure 4). A typical opening depth of 0.75 times the web depth was specified, with openings at regular 294 mm intervals, in the case of the cellular beam. If the density of steel is taken as a typical value of 7850 kg/m$^3$, the topology optimised beam weight totals 169.87 kg, whilst the cellular beam weighs 173.64 kg, hence only a 2.17% of difference.

![Figure 4 – Beam Models Compared: a) Topology Optimised Beam Web Design b) Cellular Beam Design](image)

In both cases, a stiffener was provided at mid span in order to more effectively transfer the shear forces between the top and bottom flange of the beam. This practice was proved appropriate particularly for the optimised beam, where there is a wide large web gap at the mid-span, which can cause localised stresses in the beam flanges due to high bending. In the case of the topology optimised design, it was found to be necessary to manually refine the FE mesh to prevent divergence of the solution due to highly distorted elements.

3.3.5 Results

The yield load of the cellular beam is formed to be 188.15 kN with a corresponding deflection of 23.58 mm. The yield load of the optimised beam is formed to be 207.96 kN with a corresponding deflection of 19.21 mm. The ultimate load of the optimised beam is also found to be higher than the cellular beam. Load versus mid-span deflection plots for the analyses (Figure 5) show that the optimised beam is stiffer than the cellular type beam in the linear region.
Figure 5 - Comparison of Load versus Mid-span Deflection for Cellular Beam and Topology Optimised Beam

Von Mises stress plots (Figure 6) for both of the beams, plotted at the yield load level in each case, show that despite some high localised stress concentrations in the optimised beam the stress distribution is generally more uniform. In contrast stresses in the cellular beam tend to increase significantly towards the support.

Figure 6 - Von Mises Stress Plots for Cellular and Topology Optimised Beams at Yield Load Level

It is observed that the cellular beam suffered a localised buckling type failure in the web post towards the support. The primary failure mode of the optimised beam is more difficult to determine. Localised buckling within the web depth is observed at
failure. High stress concentrations around the large central opening do, however, indicate a Vierendeel type failure. Both of these features are observed to occur concurrently.

3.4 Results Discussion

The web design of an I-section steel beam is developed using topology optimisation. When compared to a conventional cellular beam, it was found that the topology optimised beam has advantageous structural behaviour in terms of the initial stiffness, the yield load and the failure load. The investigation was therefore considered to be a proof of concept demonstrating that there are more effective ways of distributing the material within the beam web depth in order to enhance the structural performance of the section.

It is worth mentioning that the effort required to develop the topology optimised beam web design was not trivial. It would be necessary to perform the lengthy topology optimisation, geometry extraction and load verification procedure for every new section, span or load combination considered. Also, the load capacity of the optimised beam web cannot be verified using standardised design equations due to the complexity of the new geometry. Clearly identifying the failure behaviour of the optimised beam, in the absence of full scale testing, was found to be challenging.

4 Development of a Topologically Optimum Web Opening Type

In light of the results detailed above, it was concluded that the topology optimisation is a useful tool for identifying alternative improved beam web opening designs. However, when it was applied to a full length section, the resulting design was complex and difficult to apply in a general sense.

In order to establish a topologically optimum opening architecture that can be applied to all beam cross-sections and positioned at any location along the beam, a local modelling approach was taken. Rather than considering a full length beam with externally applied loading, a short section of a beam was modelled and internal shear and bending moments were applied directly to the section. This approach enables a wide variety of cross-sections to be investigated, and will result in a new opening type that could be utilised generally.

4.1 Parametric Topology Optimisation Approach

A FE model of a short beam section was developed. Loading was applied to the model in the form of bending moments and shear forces. Rigid elements were used in the application of the bending moment and shear force (Figure 7).
Three load cases were considered in the topology optimisation: bending moments only, shear forces only, and a combination of shear forces and bending moments.

A weighted compliance approach was taken when considering the three load cases in the optimisation. The weighting for each of the individual compliances was defined relative to the compliance of the plain webbed section subject to each of the load cases. The process of determining the weightings for an illustrative example is summarised in Table 1.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Moment</th>
<th>Shear</th>
<th>Moment-Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compliance of Plain Webbed Section</td>
<td>97,951</td>
<td>33,177</td>
<td>61,237</td>
</tr>
<tr>
<td>Weighting</td>
<td>0.509</td>
<td>0.172</td>
<td>0.318</td>
</tr>
</tbody>
</table>

Table 1 - Illustrative Example of Compliance Weighting Calculation

The geometric parameters varied were the depth of the section and the ratio of flange and web thickness. Full details of all of the cross-sections considered in the parametric study of the depth variation (Table 2) and the flange ratio variation (Table 3), are included.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Breadth (mm)</th>
<th>Depth (mm)</th>
<th>Flange Thickness (mm)</th>
<th>Web Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>576</td>
<td>150</td>
<td>240</td>
<td>6</td>
<td>10</td>
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<tr>
<td>648</td>
<td>168.75</td>
<td>270</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>720</td>
<td>187.5</td>
<td>300</td>
<td>6</td>
<td>10</td>
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<td>1344</td>
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<td>560</td>
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<td>10</td>
</tr>
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<td>1680</td>
<td>437.5</td>
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<td>10</td>
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<tr>
<td>2160</td>
<td>562.5</td>
<td>900</td>
<td>6</td>
<td>10</td>
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Table 2 - Sections used in Topology Optimisation Studies Considering Variation to Section Depth
<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Breadth (mm)</th>
<th>Depth (mm)</th>
<th>Flange Thickness (mm)</th>
<th>Web Thickness (mm)</th>
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<td>960</td>
<td>250</td>
<td>400</td>
<td>30</td>
<td>10</td>
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</tbody>
</table>

Table 3 - Sections used in Topology Optimisation Studies Considering Variations in Flange Thickness Ratios

4.2 Results

It should be noted that when interpreting the results of the topology optimisation studies, only the opening pattern emerging in the centre of the beam (Figure 8) is of interest in terms of developing a web opening type that could be periodically applied along the length of any beam.

![Figure 8 - Identification of Generalised Opening Type from Topology Optimisation Studies on Short Section of Beam](image)

It was found that the ratio of the flange thickness to the web thickness did not alter the resulting optimal topology, but the actual shape of the suggested web opening was altered (Figure 9). The depth of the section did have an impact on the optimal topology (Figure 10). In the depth range between 270 mm and approximately 750 mm, it was found that the optimal opening topology was consistent. For shallower beams, it was found that an elongated cross shape was suggested. In the case of beams deeper than 750 mm, extra openings were recommended by the topology optimisation results (Figure 9).

The consistency of the results in the depth range between 270 mm and 750 mm indicate that the same topology is optimal for the usual depth of beam sections that may be found in practice.
<table>
<thead>
<tr>
<th>Depth</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>200mm Deep</td>
<td>![Image]</td>
</tr>
<tr>
<td>240mm Deep</td>
<td>![Image]</td>
</tr>
<tr>
<td>270mm Deep</td>
<td>![Image]</td>
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<tr>
<td>300mm Deep</td>
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<td>400mm Deep</td>
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<td>![Image]</td>
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<tr>
<td>900mm Deep</td>
<td>![Image]</td>
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</tbody>
</table>

Figure 9 - Results of Parametric Topology Optimisation Studies Considering Variations in Beam Depth
4.3 Topologically Optimum Web Opening Concept

Based on the results of the parametric studies, an optimum web opening type has been developed (Figure 11). It should be noted that this optimum topology may be interpreted using various opening shapes (Figure 12). It is anticipated that a shape optimisation study would yield valuable information regarding the optimum shape of the openings.
The concept of this opening architecture is suggested to be topologically optimal for all beam cross sections in the depth range between 270 mm and 750 mm. In the case of beams deeper than 750 mm deep, an alternative opening type is recommended as being topologically optimal (Figure 13).

It is aforementioned that it would not be possible to create beams with the suggested web opening topologies using only the profile cutting procedure. However, it would be possible to realise such beams based on the plate assembly method of fabrication. This conforms to the current trends of the steel fabrication industry (eg. Fabsec® and ASD Westok ltd.), in which the plated beam design is now vastly specified.
5 Concluding Remarks

The application of structural topology optimisation techniques to the design of openings in the web of steel I-section beams has been investigated for the first time. A web opening design for a 5m span universal beam section was developed using the solid isotropic material with the penalisation method of topology optimisation. A geometric and material nonlinear FE analysis, using ANSYS, was employed in order to predict the load carrying capacity of the topology optimised beam. It was found that when compared to a cellular type beam, the topology optimised design had increased stiffness and yield load. Various limitations to the routine application of topology optimisation, including the complexity of the web geometry as well as the analysis difficulties, were highlighted.

A parametric topology optimisation study has been performed in order to develop a topologically optimum web opening architecture for the wide variety of beam cross sections that may be found in practice. An optimum web opening architecture has been developed (Figure 11 and Figure 12), that is effective for beams of depth between 270 mm and 750 mm. An alternative optimum web opening architecture is suggested for beams deeper than 750 mm (Figure 13). Further work is anticipated to focus on the optimisation of the shape of the openings to be employed in the topologically optimum web opening architecture. Investigations of the ultimate load carrying capacities and the failure modes of the topologically optimum web opening configurations are also to be comprehensively examined. The ultimate aim is the development of design equations that can be used in the routine design of beams employing the optimum web opening configurations.

References


