Simulation of Trajectories and Comparison of Joint Variables for Robotic Manipulator Using Multibody Dynamics (MBD)

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Abstract
This paper deals with simulation of different manipulator for a given value of joint variable and comparing workspace generated by both manipulators. In this work, the analytical model is validated and simulated using Multibody Dynamics (MBD) of HyperWorks. The objective of paper is to analyze singularities and variation of joint variable for both configurations while moving around a specified path. The link velocity is computed for both configuration and it is compared with analytical model. The trajectory used for manipulator is same and coding is done for forward and inverse kinematics using MATLAB 6.5.

Keywords: Joint Variable, MBD, Jacobian, Joint Velocity

Introduction
A robot manipulator requires the knowledge of end effectors position and orientation for the instantaneous location of each joint as well as knowledge of the joint displacement for placing the end effector in a new location. Therefore, the direct and inverse kinematics is the fundamental problem of the most importance in the robot manipulator’s position control. Certain applications like welding and assembly operation require that the end effector or tool should follow a given path. To achieve this, it is necessary to find corresponding motions of each joint, which will produce desired end effector motion.

The joint displacement that lead the end effector to a certain position and orientation T can be found by solving the kinematics model equations for unknown joint displacement and the location of the end effector is obtained by moving each joint by respective joint displacement.

For the manipulator, not only the final location of the end effector is important, but also the velocity with which the end effector would move to reach the final location is also an equally important concern. This requires the coordination of instantaneous end effector velocity and joint velocities.

The transformation from joint velocities to the end effector velocity is described by a matrix called Jacobian. The Jacobian matrix which is on manipulator configuration, is a linear mapping from velocities in joint space to velocities in Cartesian space. The inverse problem, where the joint velocities are to be determined for given end effector velocity is of practical importance and requires the inverse of the Jacobian. At certain location in joint space, the Jacobian matrix may lose rank and it may not be possible to find its inverse. These locations are referred as singular configuration.

The end effector of the manipulator is required to move in a particular path or trajectory to accomplish given task. The trajectory planning can be described as a time sequence of joint or end effector locations and derivatives of locations, which are generated by interpolation or approximation. Also the position and orientation of the end effector can be defined in a Cartesian space. The inverse kinematics problem is set to find end effector velocity and subsequent solution of joint velocities and Jacobian.

Mathematical Modeling
The articulated arm shown in Figure 1 consists of three links with rotary joints. The first link consists of rotation about vertical axis and other two links simulates the human arm with shoulder and elbow joint. The work volume of this configuration is spherically shaped.
The Table - 1 describes the D – H parameter for both manipulators.

<table>
<thead>
<tr>
<th>Joint</th>
<th>a_i</th>
<th>( \alpha_i )</th>
<th>d_i</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>90(^\circ)</td>
<td>0</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>L2</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>L3</td>
<td>0</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
</tbody>
</table>

**Forward Kinematics Equation**

\[
{}^0T_1 = \begin{bmatrix}
\cos \theta_1 & 0 & \sin \theta_1 & 0 \\
\sin \theta_1 & 0 & -\cos \theta_1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
{}^1T_2 = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 & 0 & L_2 \cos \theta_2 \\
\sin \theta_2 & \cos \theta_2 & 0 & L_2 \sin \theta_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
{}^2T_3 = \begin{bmatrix}
\cos \theta_3 & -\sin \theta_3 & 0 & L_3 \cos \theta_3 \\
\sin \theta_3 & \cos \theta_3 & 0 & L_3 \sin \theta_3 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The overall transformation matrix for the end point of the arm is,
Velocity and Jacobian Calculation

The mapping of the linear angular velocity vector, from frame \{1\} to frame \{2\}, can have reference of two frames, the frame of description and the frame of differentiation. For the manipulator, the frame of differentiation is base frame, that is, frame \{0\}. The n – DOF manipulator is an open Kinematic chain with n links. The base, link 0, is the stationary reference and velocity of all other links is defined with respect to this. The linear velocity of the i\(^{th}\) link is given by following equation:

\[
v_i = \sum_{j=1}^{n} \left( \frac{\partial {0T}_j}{\partial q_j} \right) q_i D_j
\]

The angular velocity of i\(^{th}\) link is given by following equation:

\[
\omega_i = \omega_{i-1} + \theta_{i-1}^T \omega_{i-1} \cdot q_i
\]

From the above equation, it is observed that the Cartesian velocities of the end effector are linearly related with the joint velocities. This relationship is shown as:

\[
V_e(t) = J(q) \cdot \dot{q}
\]

Where, \(V_e(t)\) = 6 X 1 Cartesian velocity vector = \(\begin{bmatrix} v \\ \omega \end{bmatrix}\)

\[J(q) = 6 \times n \text{ Manipulator Jacobian or Jacobian Matrix} = \begin{bmatrix} J_{vi} \\ J_{\omega} \end{bmatrix}\]

\(q = n \times 1 \) vector of joint velocities

Results & Discussions from Mathematical Me

The Figure 2 shows the trajectory generated by program. The Figure 3 and Figure 4 indicated velocity variation through given range of displacement.
Results & Discussion from simulation using MBD of Hyperworks

The virtual model is required to create and implement the concept. The mathematical model provides the basis for synthesizing the dimensions of the various bodies and parts to be assembled.

Looking into the type of robot and limiting displacement and velocities of various joints and links, the model can be created in any modeling software. In this work the model is created with the help of Pro/Engineer 5.0. The same is translated into *.step files and imported into Multi Body Dynamics (MBD) environment MotionView of HyperWorks to carryout kinematic and dynamic analysis.

The TABLE – 2 gives information of Points and Bodies imported from the assembly.

<table>
<thead>
<tr>
<th>POINTS</th>
<th>LX</th>
<th>LY</th>
<th>LZ</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global Origin</td>
<td>0.000000e+000</td>
<td>0.000000e+000</td>
<td>0.000000e+000</td>
<td></td>
</tr>
<tr>
<td>BASE_3.PRT CG</td>
<td>0.000000e+000</td>
<td>6.181387e+000</td>
<td>0.000000e+000</td>
<td></td>
</tr>
<tr>
<td>C1_2.PRT CG</td>
<td>0.000000e+000</td>
<td>4.500000e+001</td>
<td>-1.285693e-014</td>
<td></td>
</tr>
<tr>
<td>R1_6.PRT CG</td>
<td>0.000000e+000</td>
<td>1.132219e+002</td>
<td>-3.535995e-014</td>
<td></td>
</tr>
<tr>
<td>LINK2_2.PRT CG</td>
<td>3.301310e+001</td>
<td>1.280372e+002</td>
<td>-1.044825e+001</td>
<td></td>
</tr>
<tr>
<td>LINK_6.PRT CG</td>
<td>9.694930e+001</td>
<td>1.541275e+002</td>
<td>-3.068328e+001</td>
<td></td>
</tr>
<tr>
<td>E_F_3.PRT CG</td>
<td>1.613411e+002</td>
<td>1.824744e+002</td>
<td>-5.106250e+001</td>
<td></td>
</tr>
<tr>
<td>PRT0001_2.PRT CG</td>
<td>1.836901e+002</td>
<td>1.938735e+002</td>
<td>-5.813569e+001</td>
<td></td>
</tr>
<tr>
<td>Point_Link6_Rev</td>
<td>6.951310e+001</td>
<td>1.410372e+002</td>
<td>-1.280000e+001</td>
<td></td>
</tr>
</tbody>
</table>
The following figure shows the results obtained from Multibody dynamics (MBD).

Figure 5: Tracing of End Effector
Figure 6: Displacement of End Effector (MBD)

Figure 7: Velocity of End Effector (MBD)

Figure 8: Displacement of Link6 (MBD)
Benefits Summary
Theoretical model i.e. mathematical model is to be checked with some CAD modeling for feasibility. The kinematic and dynamic equations can provide strong basis for design of joints and numerical values can be obtained to select the actuators to drive the joints. However, it is necessary to prepare 3D CAD model and carryout Kinematic and Dynamic analysis to validate mathematical model.

The simulation of virtual ROBOT leads to actual feel of motion of all links and joints as per designed in the theoretical model. Also workspace of all the links can be plotted to analyze the maximum and minimum reach of various points on the links.

Challenges
The methodology can be extended to analyze various manipulators for the same inputs and outputs. The selection of best suited manipulator can be made by such analysis. However, the mathematical modeling and programming is required to facilitate solution for all type of configurations of robot.

Conclusions
The software solution will enable the learner to develop the concept from basic mathematics to actual prototype development to achieve objectives.

REFERENCES