Example 11 - Tensile Test

Summary
The material characterization of ductile aluminum alloy is studied. The RADIOSS material laws 2, 27 and 36 are used to reproduce the experimental data of a traction test by simulation. The work-hardening, damage and rupture of the specimen are simulated by a finite element model. The parameters of the material laws are determined to fit the experimental results. The influence of the strain rate is also studied. A strain rate filtering method is used to reduce the effect of a dynamic resolution on the simulation results.

- Law characterization
- Strain rate effect
# Law Characterization

<table>
<thead>
<tr>
<th>Title</th>
<th>Law characterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>11.1</td>
</tr>
</tbody>
</table>

**Brief Description**

Elasto-plastic material law characterization using a tensile test.

**Keywords**

- Shell element
- Johnson-Cook elasto-plastic model (/MAT/LAW2)
- Tabulated elasto-plastic (/MAT/LAW36)
- Elasto-plastic brittle (/MAT/LAW27)
- Necking point, damage model, maximum stress, failure plastic strain

**RADIOSS Options**

- Boundary conditions (/BCS)
- Imposed velocities (/IMPVEL)
- Material definition (/MAT)

**Compared to / Validation Method**

- Experimental results

**Input File**

- Law 2 Johnson Cook: `<install_directory>/demos/hwsolvers/radioss/11_Tensile_test/Law_2_Johnson-Cook/.../TENSIL2*`
- Law 27 Damage: `<install_directory>/demos/hwsolvers/radioss/11_Tensile_test/Law_27_Damage/DAMAGE*`
- Law 36 Tabulated: `<install_directory>/demos/hwsolvers/radioss/11_Tensile_test/Law_36_Tabulated/TENSI36*`

**RADIOSS Version**

44q

**Technical / Theoretical Level**

Advanced
Overview

Aim of the Problem

It is not always easy to characterize a material law for transient analysis using the experimental results of a tensile test. The purpose of this example is to introduce a method for characterizing the most commonly used RADIOSS material laws for modeling elasto-plastic material. The use of "engineering" or "true" stress-strain curves is pointed out. Damage and failure models are also introduced to better fit the experimental response.

Apart from the experimental results, the modeling of the strain rate effect on stress will be considered at the end of this example using a sensitivity study on a set of parameters for Johnson-Cook’s model.

Physical Problem Description

Traction is applied to a specimen. A quarter of the specimen is modeled using symmetrical conditions. The material to be characterized is 6063 T7 Aluminum. A velocity is imposed at the left-end.

Units: mm, ms, g, N, MPa.

The material undergoes isotropic elasto-plastic behavior which can be reproduced by a Johnson-Cook model with or without damage (/MAT/LAW27 and /MAT/LAW2, respectively). The tabulated material law (/MAT/LAW36) is also studied.

Fig 1: Geometry of the tensile specimen (One quarter of the specimen is modeled).

Fig 2: Experimental results of the tensile test: engineering stress vs. engineering strain.
Analysis, Assumptions and Modeling Description

Modeling Methodology
The mesh is shown in Fig 3. The average element size is about 2 mm. There are 201 4-node shells and one 3-node shell.

The shell properties are:

• 5 integration points (progressive plastification).
• Belytschko elasto-plastic hourglass formulation ($I_{\text{plas}} = 3$).
• Iterative plasticity for plane stress (Newton-Raphson method; $I_{\text{plas}} = 1$).
• Thickness changes are taken into account in stress computation ($I_{\text{thick}} = 1$).
• Initial thickness is uniform, equal to 1.7 mm.

![Node 1]

Fig 3: Mesh of the specimen.

• Node number 54 was renamed "Node 1" to be compliant with the Time History.
For node 54, only displacements in the x-direction (variable DX) are saved.

![Section 1 and Section 2]

Fig 4: Sections saved for Time History.

For both sections, the variables FN and FTX, are saved; thus the following variables will be available in /TH/SECTIO: FNX, FNY, FNZ (saved using "FN"), FTX.

Engineering strains will be obtained by dividing the displacement of node 1 with the distance up to the symmetry axis (75 mm). Engineering stresses will be obtained by dividing the force through section 1 with its initial surface (10.5 mm²). Therefore, the results shown correspond to the engineering stress as a function of the engineering strain, equivalent to the force variation compared to displacement (similar curve shape).
RADIOSS Options Used

An imposed velocity of -1.0 m/s in the x-direction is applied to the nodes, shown below (abscissa less or equal to 25 mm). The displacement is proportional to time.

![Imposed velocities](image)

**Fig 5: Imposed velocities**

![Variation of node 1 x-displacement in relation to time](image)

**Fig 6: Variation of node 1 x-displacement in relation to time.**

Only one quarter of the specimen is modeled to limit the model size and to eliminate the rigid body motions. Symmetry planes are defined along axis x = 100 mm and axis y = 0. Note that two boundary conditions cannot be applied to the same node 13 (corner).

![Boundary conditions](image)

**Fig 7: Boundary conditions**

The lower side is fixed in Y, Z translations and X, Y, Z rotations.
The right side is fixed in X, Z translations and X, Y, Z rotations; the node in the corner is completely fixed.

**Characterization of the Material Law**

There are two steps to characterize the material law:
- Transform the engineering stress versus engineering strain curve into a true stress versus true strain curve (this step applies to any material law).
- Extract the main parameters from the true stress versus true strain curve, to define the material law (Johnson-Cook law and material coefficients for /MAT/LAW2 or the yield curve definition for /MAT/LAW36).

**True stress/true strain curve**

Engineering strains are computed using the following relationship:

\[
\varepsilon_e = \frac{\Delta l}{l_0}
\]

And true strains are computed with the relationship:

\[
\varepsilon_n = \ln\left(1 + \frac{\Delta l}{l_0}\right)
\]

Both strains, therefore, are linked together by:

\[
\varepsilon_n = \ln\left(1 + \varepsilon_e\right)
\]

Engineering stresses are measured by dividing the force through one section with the initial section. True stresses are measured by dividing the force with the true deformed section:

\[
\sigma_e = \frac{F}{S_0} \quad \sigma_n = \frac{F}{S}
\]

Thus, to compute true stresses, the surface variation must be taken into account. Assuming that Poisson's coefficient is 0.5 during plastic deformation, the true surface in mono-axial traction is:

\[
S = S_0 \exp\left(-\varepsilon_n\right)
\]

Thus, the relationship between true and engineering stresses is:

\[
\sigma_n = \sigma_e \exp\left(-\varepsilon_e\right)
\]

**Characterization of the Material Law**

The characterization will be made for /MAT/LAW2 (Johnson-Cook elasto-plastic), /MAT/LAW27 (elasto-plastic with damaged model) and /MAT/LAW36 (tabulated elasto-plastic). For each of the material laws, the yield stress and Young's modulus are determined from the curve.

The plastic strain can be defined as:

\[
\varepsilon_p = \varepsilon_e - \frac{\sigma_e}{E}
\]

An important point to be characterized on the curve is the necking point, where the slope of the force versus the displacement curve is equal to 0, and where the following relationships apply:

\[
\frac{\partial \sigma_n}{\partial \varepsilon_n} \approx \sigma_n \quad \frac{\partial F}{\partial \delta} = 0
\]
Fig 8: Guidelines for necking point.

Table 1: Equations used for analysis

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Generic Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering stress</td>
<td>$\sigma_e = \frac{F}{S_0}$</td>
</tr>
<tr>
<td>Engineering strain</td>
<td>$\varepsilon_e = \frac{\Delta l}{l_0}$</td>
</tr>
<tr>
<td>True stress</td>
<td>$\sigma_t = \sigma_e \exp(\varepsilon_t)$</td>
</tr>
<tr>
<td>True strain</td>
<td>$\varepsilon_t = \ln(1 + \varepsilon_e)$</td>
</tr>
<tr>
<td>True strain rate</td>
<td>$\dot{\varepsilon}<em>t = \frac{\Delta \varepsilon</em>{pl}}{\Delta t}$</td>
</tr>
</tbody>
</table>
Simulation Results and Conclusions

Experimental Results

An experiment designed by the "Norwegian Institute of Technology" as part of an EC-financed program, "Calibration of Impact Rigs for Dynamic Crash Testing" is used. The following curve was obtained from the experiment:

![Engineering stress versus engineering strain curve](image)

It is estimated that the necking point occurs between 6% and 8% (engineering strain). After analyzing the experimental data, the first point satisfying the necking condition is at 6.68%.

![Comparison between engineering and true curves](image)

Engineering formulation is converted into true formulation using the relationship: \( \sigma_t = \sigma_e \exp(\varepsilon_e) \)
The true stress curve is higher than the engineering stress curve, as it takes into account the decrease in the specimen cross-section.

**Law 2: Elasto-plastic Material Law Using the Johnson-Cook Model**

**Johnson-Cook Material Coefficients**

The stress versus plastic strain law is: \( \sigma_r = \sigma_r \exp(\epsilon_r) \) (Johnson-Cook model)

where, \( \sigma \) is the yield stress and is read from the experimental curve and then converted into true stress.

To compute \( b \) and \( n \), two states are needed. This leads to the following formulas for \( b \) and \( n \):

\[
\begin{align*}
    n & = \frac{\ln((\sigma_1 - \sigma) / (\sigma_2 - \sigma))}{\ln(\epsilon_1 / \epsilon_2)} \\
    b & = \frac{\sigma_1 - \sigma}{(\epsilon_1)^n}
\end{align*}
\]

The first point is chosen at the necking point, then \( b \) and \( n \) are computed for each other point of the curve and averaged out since the results tend to differ depending on the point chosen.

**Characterization up to the Necking Point**

The first stage when determining the material model is to obtain Johnson-Cook’s coefficients. Neither the maximum stress, nor the failure plastic strain effects are taken into account here (set at zero).

The values of coefficients are chosen so that the model adapts to the test data.

![Diagram](image)

**Fig 11:** Variation of the engineering stress/strain according to Johnson-Cook’s model adapted to the test.

The material coefficients used for Law 2 are:

- Initial density: \( 2.7 \times 10^3 \) g/mm\(^3\)
- Yield stress: 90.27 MPa
- Poisson’s ratio: 0.33
- Hardening parameter: 223.14 MPa
- Young’s modulus: 60400 MPa
- Hardening exponent: 0.375

Figure 12 compares the yield curve defined using the Johnson-Cook model with the one extracted from experimental data.
The true stress – true strain relationship can be described by:

$$\sigma = 90.27 + 223.14 \sigma_{pl}^{0.375}$$

The engineering stress deviations between experiment and simulation are described in the table below:

<table>
<thead>
<tr>
<th>Engineering strain</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.067</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation</td>
<td>7.9%</td>
<td>4.8%</td>
<td>1.8%</td>
<td>1.1%</td>
<td>1%</td>
<td>1.8%</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

Comparison is performed up to the necking point (engineering strain = 6.68%) because after this state, a rapid decrease in the engineering stresses occurs in the specimen. The rupture sequence is simulated in the following paragraphs. Results using Law 2 remain within 8% of the experimental curve.

The curve could be improved by slightly adjusting some of the values. The purpose of this test is to propose a method for deducing material law parameters using a tensile test.

**Beginning of the Necking Point**

**Necking Point Simulation**

The Johnson-Cook model previously defined corresponds to the experimental results up to the necking point. However, the slope of the numerical response does not enable the necking point to start at the strain value observed experimentally.

The necking point is characterized by the slope value of the true stress versus the true strain curve, which must be approximately equal to the true stress. The necking point numerically appears by continuing simulation until the condition on the slope is observed.

The results are obtained using the Johnson-Cook model 1:

$$\sigma = 90.27 + 223.14 \sigma_{pl}^{0.375}$$
The necking point can be simulated, either by adjusting the Johnson-Cook coefficients to obtain an accurate slope, or by compelling curve with a maximum stress.

**Simulation of the Slope Near the Necking Point**

By implementing an energy approach, the hardening curve can be modified to achieve an engineering curve which resembles a horizontal asymptote near the necking point with the purpose of simulating the behavior of the curve as observed in the test.
The Johnson-Cook coefficients used to describe the physical slope are:

Yield stress: 79 MPa
Hardening parameter: 133 MPa
Hardening exponent: 0.17

For this model, the new true stress / true strain relationship is:

$$\sigma = 79 + 133\varepsilon^{0.17}$$ (Johnson-Cook model 2)

The results obtained with those coefficients are provided below.

Figure 15 compares the Johnson-Cook model 3 with the experiment:

![Graph showing adjusted engineering stress/strain curve to model the beginning of the necking point.](fig15)

Fig 15: Adjusted engineering stress/strain curve to model the beginning of the necking point.

The shape of the yield curve versus the experimental data is depicted in Fig 16.
The necking point is defined as
\[
\frac{\partial \sigma_{tr}}{\partial \varepsilon_{tr}} \approx \sigma_{tr}
\]
This condition is characterized by the intersection of the true stress versus the true strain curve with its derivate.

**Beginning of the Necking Point Using a Maximum Stress Limit, \( \sigma_{\text{max}} \)**

For this test, the Johnson-Cook coefficients input are those set in characterization up to the necking point, the failure effect not being taken into account (the failure plastic strain is set to zero). The beginning of the necking point is set using the choice of a maximum stress value. In comparison to the experimental results (see Fig 10), the necking point is well defined for a maximum stress set at 175 MPa. The limit in stress appears on the von Mises stress versus true strain curve on elements where the necking point occurs.

The maximum true stress manages the beginning of the necking, as shown below:
\[ \sigma_e = \sigma_T \exp(-\varepsilon_{tr}) \]

Hence:
when \( \varepsilon_{tr} \) increases
\( \sigma_e \) decreases

Necking point: engineering strain = 6.68%
Maximum stress $\sigma_{\text{max}}$ is reached for von Mises stress on shells where the necking begins. To avoid overly-high stresses after the necking point, a maximum stress factor must be set approximately equal to the true necking point stress.

The following curves show the evolution of the von Mises stress versus the true strain shell at two characteristic locations of the specimen (3b and 3a in Fig 20):

![Fig 20: von Mises stress curve with a maximum stress limit.](image)

The beginning of the necking point is observed following the point where the stress is equal to stress versus strain derivative $\sigma = \frac{\partial \sigma}{\partial \varepsilon}$.

![Fig 21: Yield curve with maximum stress.](image)

The yield curve is described by:

$$\sigma = \min(\sigma_{\text{max}}, (a + b\varepsilon_p))$$

The derivative of the stress is very sensitive and strongly depends on the yield curve definition. Thus, introducing the necking point into the simulation is very delicate (a small change can result in many variations). The necking point should first begin on a given element for numerical reasons. The preferred beginning of necking is addressed below.
**Preferred Beginning of the Necking Point**

Experimentally, the beginning of the necking point can appear anywhere on the specimen. The beginning of the necking point should preferably be located on the right end elements in order to propose a methodology for this quasi-static test. If the model only uses a quarter part of the specimen, the necking point is found on elements 30, 125 and 78.

The beginning of the necking point is physically and numerically sensitive and can be initiated on the right elements by changing a few of the coordinates along the Y-axis of the node in the right corner (node 16) in order to decrease the cross-section and privilege the necking point in this zone. Changing the node position by 0.01 mm is enough for achieving the preferential beginning of the necking point.

![Fig 22: Node 16 to be moved.](image)

A second approach also enables the necking point to be triggered on the right end side by defining an extra part, including shells 3, 11 and 4 by using a maximum stress slightly lower than the remaining part, in order to initiate the necking point locally since the necking point stress is first reached in the elements having the lowest maximum stress value, that is shells 3, 11 and 4. This method, based on material properties, is quite appropriate for demonstrating the characterization of a material law and will thus be used in the continuation of the example.

![Model with one part: maximum stress set at 175 MPa. Beginning necking point: shells 30, 125 and 78.](image)

![Model with two parts: maximum stress set at 174 MPa for elements on right-extremity and 175 MPa for other elements. Beginning necking point: shells 3, 11 and 4.](image)

![Fig 23: Localization of the beginning of the necking point according to the models using $\sigma_{\text{max}}$.](image)

The material is described as Johnson-Cook model 1:

$$\sigma = 90.27 + 223.14 \varepsilon_{\text{pl}}^{0.375}$$

$$\sigma_{\text{max}} = 174 / 175 \text{ MPa}$$

The following curves indicate the variation of the engineering stress versus the engineering strain according to the beginning of the necking point zone and in comparison to the experiment.
Fig 24: Engineering stress/strain curve for each starting necking point location.

There is a fast decrease in the engineering stress after the right-end necking point. The necking point, due to the boundary conditions of the y-symmetry plane (y-translation d.o.f. released), becomes more pronounced.

Note that the variations in the section where the necking point is found are quite similar up to the necking point. After such point, there is a sharp surface decrease for the right-end necking point, contrary to the second case where the surface decrease is more moderate.
Improvement of the Elements’ Contribution During the Necking Point Sequence

In order to simulate physically the contribution of each element in the necking point, it is advisable to adjust the curve by varying the Johnson-Cook coefficients in order to increase the intensity of stress at the necking point. The main result is no longer the variation of the stress/strain curve but rather the surface under the curve which characterizes the energy dissipated during the test. This energy-based approach is relevant for crash tests since the final assessment is often more significant than how it was achieved.

\[
\text{Energy} = \int \sigma \varepsilon \, d \varepsilon
\]
The following graph compares the new yield curve with experimental data:

Material is described in the Johnson-Cook coefficients are:

**Johnson Cook model 3:**

\[
\sigma = 50 + 350 \varepsilon^{0.38}
\]

(true stress/strain)

Yield stress = 50 MPa
Hardening parameter = 350 MPa
Hardening exponent = 0.38
Maximum stress is set to 189 or 190 MPa (according to the parts)

The results of adjustment to the Johnson-Cook coefficients are depicted below:
As the necking point progresses, more physical results are obtained due to the new input data of the material law coefficients having a better element contribution.

\[ \sigma = 90.27 + 223.14 \varepsilon_{pl}^{0.375} \quad \text{Law 2, without yield curve change (model 1)} \]

\[ \sigma = 50 + 350\varepsilon_{pl}^{0.38} \quad \text{Law 2, with modified yield curve (model 3)} \]

Fig 28: Shell contribution during the necking point sequence (von Mises stress).

Damage Modeling with Plastic Strain Failure

The elasto-plastic model of Johnson-Cook is used until failure, which is simulated using a plastic strain failure option. The element is deleted if the plastic strain reaches a user defined value \( \varepsilon_{\text{max}} \). This damage model shows good stability. A maximum plastic strain is defined for each Johnson-Cook model:
Fig 30: $\varepsilon_{\text{max}} = 75\%$; yield curve close to experimental data:

$$
\sigma = 90.27 + 223.14 \varepsilon_{\text{pl}}^{0.375}
$$

From the Johnson-Cook model 1

Fig 31: $\varepsilon_{\text{max}} = 47\%$; yield curve adjusted with respect to lower stresses:

$$
\sigma = 70 + 133 \varepsilon_{\text{pl}}^{0.17}
$$

From the Johnson-Cook model 2
Failure is reached for relatively high true strains.

**Law 27: Elasto-plastic Material Law with Model Damage**

Law 27 is used to simulate material damage following a Johnson-Cook plasticity law. Thus, model damage is associated with the previous law in order to take account of failure.

The damage parameters are:

- Tensile rupture strain $\varepsilon_t$: damage starts if the highest principal strain reaches this tension value.
- Maximum strain $\varepsilon_m$: the element is damaged if the highest principal strain is above the tension value.
- The element is not deleted.
- Maximum damage factors $\sigma_{\text{max}}$: this value should be kept at its default value (0.999).
- Failure strain $\varepsilon_f$: the element is deleted if the highest principal strain reaches the tension value.

\[
\sigma = 50 + 350\varepsilon_{\text{pl}}^{0.38}
\]

Fig 33: Stress/strain curve for damage affected material.

The following graphs display the results obtained using the material coefficients of two previous Johnson-Cook models. Damage parameters complete those models.
Damage Model A

Engineering stress/strain

![Engineering stress/strain graph]

Damage model:
- $t_1 = 0.16$
- $m_1 = 0.72$
- $d_{\text{max}} = 0.999$
- $f_{\text{max}} = 16$

Johnson-Cook model:
- $\sigma_{\text{max}} = 175$ MPa

Von Mises stress

![Von Mises stress graph]

Damage model: $\varepsilon_{\text{f}} = 0.16$ ; $\varepsilon_{\text{m}} = 0.72$ ; $d_{\text{max}} = 0.999$ ; $\varepsilon_1 = 1$ ; $\varepsilon_{\text{max}} = 16$

Johnson-Cook model:
- $\sigma = 90.27 + 223.14 \varepsilon_{\text{pl}}^{0.375}$

Damage Model B
Damage model: $t_1 = 0.16; m_1 = 0.45; d_{\text{max}} = 0.999; f = 1; \sigma_{\text{max}} = 16$

Johnson-Cook model: $\sigma = 50 + 350 \varepsilon_{\text{pl}}^{0.38}$
Law 36: Tabulated Elasto-plastic Law

This is a tabulated law; therefore, the true stress versus plastic strain function can be directly used. The rupture phase can be simulated by adding points to this hardening function.

<table>
<thead>
<tr>
<th>Plastic strain</th>
<th>True stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90.</td>
</tr>
<tr>
<td>0.00048</td>
<td>99.13</td>
</tr>
<tr>
<td>0.00111</td>
<td>103.6</td>
</tr>
<tr>
<td>0.00321</td>
<td>106.02</td>
</tr>
<tr>
<td>0.00478</td>
<td>112.98</td>
</tr>
<tr>
<td>0.01062</td>
<td>125.1</td>
</tr>
<tr>
<td>0.019589</td>
<td>139.926</td>
</tr>
<tr>
<td>0.031332</td>
<td>152.5</td>
</tr>
<tr>
<td>0.03895</td>
<td>159.</td>
</tr>
<tr>
<td>0.050926</td>
<td>167.</td>
</tr>
<tr>
<td>0.063808</td>
<td>172.</td>
</tr>
<tr>
<td>0.074123</td>
<td>174.7</td>
</tr>
<tr>
<td>0.081827</td>
<td>176.3</td>
</tr>
<tr>
<td>0.088427</td>
<td>177.6</td>
</tr>
<tr>
<td>0.0935</td>
<td>178.8</td>
</tr>
<tr>
<td>0.0955</td>
<td>179.2</td>
</tr>
<tr>
<td>0.09675</td>
<td>179.1</td>
</tr>
<tr>
<td>0.099</td>
<td>178.6</td>
</tr>
<tr>
<td>0.102</td>
<td>178.5</td>
</tr>
<tr>
<td>0.105</td>
<td>177.8</td>
</tr>
</tbody>
</table>

Fig 34: Hardening function defined in law 36 to obtain the results below.

Fig 35: Results obtained with tabulated law 36.
The hardening curve has to be defined with precision around the necking point while the decrease of the curve is very sensitive to its adjustment. In order to improve the modeling of the necking point, two points can be interpolated, one "just before" the necking point, and one "just after" with the slope between those two points equal to the necking point stress.
Strain Rate Effect

**Title**
Strain rate effect

**Number**
11.2

**Brief Description**
The strain rate effect is taken into account, using filtering (cut-off frequency).

**Keywords**
- Shell element
- Johnson-Cook elasto-plastic model (/MAT/LAW2)
- Engineering strain / stress, strain rate effect, filtering

**RADIOSS Options**
- Boundary conditions (/BCS)
- Imposed velocities (/IMPVEL)

**Input File**
Time_History_files:
<install_directory>/demos/hwsolvers/radioss/11_Tensile_Test/TENSILET01

**RADIOSS Version**
44q

**Technical / Theoretical Level**
Advanced

**Strain Rate Effect and Strain Rate Filtering (Cut-off Frequency)**
In this additional study, the Johnson-Cook model is used to study the strain rate influence on stress with or without filtering. There is no comparison with the experiment data in this section. The study of sensitivity will be performed up to the beginning of the necking point.

- Stress-strain relationship:

  The Johnson-Cook plasticity model will take into account the strain rate effect on the elasto-plastic material behavior in order to improve the quality of simulation.

  The law reads as follows:
\[ \sigma = \left( a + b e_p^n \right) \left( 1 + c \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \]

where:
- \( \dot{\varepsilon} \): is the strain rate
- \( \dot{\varepsilon}_0 \): is the reference strain
- \( \varepsilon_p \): is the plastic strain (true strain)
- \( c \): is the strain rate coefficient

The two optional inputs, strain rate coefficient and reference strain rate must be defined for the material. The purpose of the sensitivity study is to illustrate the influence of material parameters.

For further explanations about the Johnson-Cook model, refer to "Elasto-plasticity of Isotropic Materials" in the RADIOSS Theory Manual.

**Strain Rate Filtering**

Because of the numerical application of dynamic loadings, the strain rates cause high frequency vibrations, which are not physical; thus the stress/strain curves look "noisy". To obtain smooth results, the strain rate filtering option will allow the reduction of those oscillations by removing the high frequency vibrations. A cut-off frequency for strain rate filtering (\( F_{cut} \)) is used since its value has to be smaller than half of the sampling frequency (\( 1/\Delta t \)).

In this example, \( \Delta t = 0.2163 \times 10^{-3} \) ms.

The constants \( a \), \( b \) and \( n \) which define the shape of the stress/strain curve are:

\[ \sigma = (90.27 + 223.14 \varepsilon_p^{0.375}) (1 + C \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)) \]

- \( a = 90.27 \) MPa
- \( b = 223.14 \) MPa
- \( n = 0.375 \)

The results are reported in the following tables.
Strain Rate Effect - Plasticity Model: Johnson-Cook

The influence of the strain rate and stress smoothing are shown below (with $\dot{\varepsilon} = 5 \times 10^3$ ms$^{-1}$ and $c = 0.1$):

**Stress Comparison**

![Stress Comparison Graph](image)

1. V Strain rate effect; without smoothing
2. O Strain rate effect; cut-off frequency: 10 kHz
3. © No strain rate effect
Influence of the Cut-off Frequency for Smoothing

![Graph showing the influence of cut-off frequency on engineering stress and strain.](image)

- **1.** Cut-off frequency: 50 kHz
- **2.** Cut-off frequency: 25 kHz
- **3.** Cut-off frequency: 10 kHz
- **4.** Cut-off frequency: 5 kHz
The following results show the effect of the reference strain rate, \( \varepsilon \), and strain rate coefficient, \( c \):

**Influence of the Reference Strain Rate \( \varepsilon \)**

(c = 0.1 and \( F_{cut} = 10 \text{ kHz} \))

<table>
<thead>
<tr>
<th>Reference Strain Rate</th>
<th>Engineering Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005 m/s^{-1}</td>
<td>50.00</td>
</tr>
<tr>
<td>0.01 m/s^{-1}</td>
<td>100.00</td>
</tr>
<tr>
<td>0.02 m/s^{-1}</td>
<td>150.00</td>
</tr>
</tbody>
</table>
Results are smoothed with correct cut-off frequencies.

Figure 36 compares the distribution of the first principal strain rate in the specimen, with and without strain rate filtering.

Fig 36: First principal strain rate comparison at time $t=4$ ms.
A more physical strain rate distribution is achieved by filtering. Moreover, such results show spatial oscillations when not damped by filtering. The explicit scheme is an element-by-element method and the local treatment of temporal oscillations puts spatial oscillations into the model.

• **Strain rate coefficient c influence:**
  If c is set to zero, the strain rate effect is not taken into account. This coefficient affects the yield stress and it slightly translates curves in the plastic region. It must be adjusted in accordance with the reference strain rate.

• **Reference strain rate influence:**
  If the strain rate is lower than the reference strain rate, there is no strain rate effect. Therefore, the lower the reference strain rate, the more the effect will be emphasized. The effect appears as a translation of the curve towards higher stresses. An increase in the flow stress using an increasing reference strain rate is observed.

• **Cut-off frequency influence:**
  The cut-off frequency must not be set higher than half of the sampling frequency. Smoothing is improved as the cut-off frequency comes closer to a particular value and the convergence of the curve until a smoothing curve can be observed. A high-reference strain rate requires low cut-off frequencies.

**Conclusion**
A tensile test is simulated using several material laws in RADIOSS. A method is set up to correspond to the material parameters in the Johnson-Cook model. The rupture phase is very sensitive and the simulation results strongly depend upon the starting point for necking. The point-by-point definition of the hardening curve in law 36 enables to bypass the adaptation difficulties when using the Johnson-Cook model. However, the results following the necking point are very sensitive to the position of points defining the hardening curve.

A method to filter the strain rate is also demonstrated. The method can be generalized to the industrial cases.