Nonlinear Sloshing Simulation under Dynamic Excitations

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Abstract: This paper simulated non-linear sloshing effects under harmonic and seismic excitations. Two separate approaches were used in the simulation, one under computational fluid dynamics (CFD) in AcuSolve and the other through finite element analysis (FEA) method using the Arbitrary Lagrangian Eulerian (ALE) formulation plus hydrodynamic biomaterial liquid gas materials in RADIOSS. The study first investigated two dimensional (2D) sloshing problems under harmonic excitations. Through calibration studies in standard rectangular tanks, the two approaches demonstrated reasonable agreement with both the analytical solution and numerical results published by other researchers. The study was then extended to more complicated three dimensional (3D) sloshing problems, with the fluid-structure interaction (FSI) considered. The simulation well reflected the sloshing behaviors in a steel tank subject to given seismic excitations and provided available prediction for structural performance. The obtained results show that the used method is helpful for seismic design of possible LNG tanks

Key words: nonlinear sloshing, fluid structure interaction, water tank, dynamic excitations, AcuSolve, RADIOSS

1. Introduction

“Sloshing motion” occurs when liquid in a container is subject to external excitations. Especially violent sloshing waves may occur due to resonance when the excitation frequency approach to the natural frequencies of the liquid which causes damage to the tank. On the other hand, the high demanding for natural gas in the international market has become driving forces for large size liquefied natural gas (LNG) tanks and supersize LNG carriers. The increase of size makes the tanks more vulnerable under sloshing conditions unless the tank was designed properly. With reference to several strong earthquakes occurred in 1990s, there are many reports on damages of liquid storage tanks. The common damages were the diamond-shape buckling [1], elephant-foot buckling, roof buckling and tank uplifting [2], caused by hydrodynamic pressures on the tank walls. As a result, a more accurate consideration of the sloshing effect in the seismic analysis and/or other dynamic analysis is important for designing liquid storage tanks. This is the background of the study of the paper.

The earliest study of sloshing problems started from theoretical investigations, in which analytical solution is available for low-magnitude sloshing in containers with simple geometry. However, the linear wave theory cannot overcome its limitation when extended to violent sloshing in complex geometries [3], so intensive studies were conducted for the non-linear sloshing behaviors through numerical approaches, such as the volume of fluid (VOF) technique [4], the SPH method [5], the multimodal or pseudospectral [6], etc. The studied targets also developed from 2D planar problem into seismic induced sloshing problems of rectangular or cylindrical tanks in the 3D space.

In this paper, both 2D sloshing problems and 3D ones are simulated, using CFD and FEA approaches respectively. The study first focused on typical 2D water sloshing problems under harmonic excitation; the results from both approaches reached reasonable agreement compared with analytical solutions and published experimental or numerical data. The study was then, through the FE approach, extended to 3D practical cases by simulating the FSI in a steel water tank subject to seismic excitation. The simulation well reflected the physical characteristics of the problem and the structural responses provided useful information for structural design of similar structures.
2. Analytical results

2.1 Linear theory for harmonic excitation

Consider a tank subject to a horizontal harmonic displacement excitation \( X(t) = D \sin(\omega t) \), where \( D \) is the amplitude of the motion, \( \omega \) is the exciting frequency and \( t \) is given time. Then, as extensively introduced by other researchers, the velocity potential relationship can be written below

\[
\nabla^2 \phi = 0 \tag{1}
\]

where \( \phi \) is the velocity potential function. Due to the slip boundary condition, the velocity normal to the boundary is zero. For instance, at \( x = \pm a \),

\[
\frac{\partial \phi}{\partial x} = 0 \tag{2}
\]

At \( y = -h \), there is

\[
\frac{\partial \phi}{\partial y} = 0 \tag{3}
\]

where \( a \) is the half length of the tank and \( h \) is the water depth. The dynamic free surface boundary condition of the problem is

\[
\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial y} - g \eta - \frac{1}{2} \left[ \nabla \phi \times \nabla \phi \right] - x \ddot{X}(t) \tag{4}
\]

The kinetic free surface boundary condition is

\[
\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial x} \tag{5}
\]

The natural frequency of sloshing in the tank \(^7\) is

\[
\omega_{nm}^2 = g \pi \sqrt{\left( \frac{m^2}{4a^2} + \frac{n^2}{4b^2} \right) \tanh \left( \pi \sqrt{\frac{m^2}{4a^2} + \frac{n^2}{4b^2}} d \right)} \tag{6}
\]

Where \( m \) is the \( m \)th mode in \( x \) direction, and \( n \) implies the \( n \)th mode in the \( y \) direction. For the fundamental frequency in the \( x \) direction, with substitution of \( m = 1 \) and \( n = 0 \), there is

\[
\omega_b = \sqrt{\frac{g \pi}{2a} \tanh \left( \frac{\pi d}{2a} \right)} \tag{7}
\]

As long as the linear solution is considered, free surface boundary condition should be imposed at the equilibrium location of the free surface, with the second-order or other higher-order differential terms ignored. Based on the solution of Wu \(^8\), the sloshing wave elevation can be given as
\[
\eta = \frac{D}{g} x \omega^2 \sin \omega t + \frac{D}{g} \left( \frac{8a^2}{\pi^2} \right) \sin \frac{\pi x}{2a} \left( \frac{\omega^4}{\omega_0^4 - \omega^2} \sin \omega t - \frac{\omega_0^3}{\omega_0^4 - \omega^2} \sin \omega t \right) 
\]

(8)

Where the first term relates to the excitation frequency \(\omega\) and the second term relates to the fundamental frequency \(\omega_0\). The free surface elevation at the tank wall, for example \(x = a\), can be written as:

\[
\eta = \frac{D}{g} x \omega^2 \sin \omega t + \left( \frac{8a^2}{g \pi^2 (\omega_0^4 - \omega^2)} \right) \omega_0^3 \sin \omega t - \frac{\omega_0^3}{\omega_0^4 - \omega^2} \sin \omega t 
\]

(9)

Eqs (7) and (9) will be used next to verify the results obtained from numerical methods in Sections 3.1 and 3.2.

2.2 Code based formula for hydrodynamic pressure under earthquakes

The hydrodynamic pressure is composed of two parts, the convective component due to water sloshing and the impulsive component due to inertia force from water. There are code-based guidelines for estimating them. For example, IITK-GSDMA \([9]\) gives frequencies for convective and impulsive frequencies as below:

\[
T_C = C_C \sqrt{\frac{D}{g}} 
\]

(10)

\[
T_i = C_i \frac{\sqrt{\rho}}{\mu / D} \sqrt{E} 
\]

(11)

Where \(C_C\) and \(C_i\) are given as:

\[
C_C = \frac{2\pi}{\sqrt{3.68 \tanh(3.68h / D)}} 
\]

(12)

\[
C_i = \frac{1}{\sqrt{h / D \left(0.46 - 0.3h / D + 0.067(h / D)^2\right)}} 
\]

(13)

The expression for the code-based hydrodynamic pressure is more complicated and is not provided in this paper. Eqs (12) and (13) will be used to verify the numerical results in Section 3.3.

3. Nonlinear sloshing simulation through numerical methods

When the magnitude of the sloshing waves become large, the higher-order components of the solution of potential equation should be included, and this is normally considered by numerical methods.

3.1 2D sloshing in rectangular tank (harmonic excitation) by the CFD approach

As shown in Figure 1, the studied tank has dimension of 0.96m (length) x 0.40 (width) x 1.00m (height), with filled ratio of 62.4%. In the simulation (by AcuSolve), the tank is considered rigid and provides only boundary constraints to the water it contains; the interface between the water and the rigid steel wall is
realized by guide surface. In order to minimize computation time, only one layer of element in the out-plane direction is considered. The magnitude of displacement excitation $A_{\text{max}}$ is 0.005m and the excitation frequency is 4.409 (rad/s), 5.550 (rad/s) and 6.221 (rad/s), corresponding to the exciting frequency below, right at and above the natural frequency respectively. The numerical model of this case is given in Figure 2.

![Figure 1 Model Sketch of case 1](image1.png)

![Figure 2 CFD Model](image2.png)

Figure 3 gives the power spectrum density (PSD) of sloshing wave elevation near the left boundary of the tank. According to the figure, one peak occurs at $f = 0.702 \, \text{Hz} \, (\omega_0 = 4.409 \, \text{rad/s})$, corresponding to the exciting frequency; the other occurs at $f = 0.879 \, \text{Hz} \, (\omega_0 = 5.531 \, \text{rad/s})$, which is right the fundamental natural frequency estimated through Eq (7). This proves Eqs (8) and (9) that the sloshing is caused by enforced vibration at the excitation frequency and the free vibration at the natural vibration.

![Figure 3 PSD of sloshing elevation at the left tank side](image3.png)

Figure 4 gives time history of sloshing elevation at the left tank boundary, obtained by Eq (9) and the CFD simulation respectively. The figure shows good agreement between the two in the non-resonant zones (Figure 4-a and Figure 4-b). In the resonant zone (Figure 4-c), the sloshing becomes violent and divergent, and obvious disagreement between the analytical and numerical results is observed. Different from equal positive and negative magnitudes in the analytical solution, the simulation shows significantly bigger positive wave elevation. Such a fact is consistent with the experimental results reported by Mohammad [10] and he explained this was due to nonlinear effect contributed by the higher-order component in the solution of the potential equation.

![Figure 4 Time history of sloshing wave elevation at the left tank boundary](image4.png)
Figure 5 further compares the water profile at the given time between the CFD simulation and the experimental results by Mahammod when resonance occurs. The profile also demonstrates perfect match between the two. All these results demonstrate the accuracy of the current simulation over the linear theory method and its reliability under even violent sloshing against experimental data.

3.2 2D sloshing in rectangular tank (harmonic excitation) by FEA approach considering two phase flow

As shown in Figure 6, the second studied case has tank dimension of 0.6m x 0.3m and filling ratio of 33%, which is an example comprehensively studied by Hitoshi using the SPH method [5]. Harmonic displacement excitation with the magnitude of 0.05m and exciting period T of 1.3s and 1.5s are considered. The simulation is conducted under the FEA package RADIOSS. ALE formulation, which can generate new undistorted mesh for large deformation problem, and hydrodynamic bi-material law for liquid and gas (Law 37, BIPHASE) are used to model the interaction between water and air. For simplicity, tank wall is not simulated but is treated as rigid boundary conditions instead. The maximum of displacement excitation is 0.05m and The FE element mode is given in Figure 7.

Figure 8 gives the sloshing water profile of experimental results by Kishev [11], the SPH numerical results by Gotoh [5] and the simulation results in this paper, with excitation period T of 1.3 s. It can be found that the current simulation gives reasonable match on water profile compared with the published data. However, a phase delay of a period T is also observed and this is possibility due to the phase angle difference when converting the excitation from displacement into acceleration.
Figure 8 Comparison of water profiles from different approaches

Figure 9 gives pressure time history measured at the left tank boundary shown in Figure 6, at excitation period $T$ of 1.5s. It can be found that the current simulation result (Figure 9-b) shows same trend and similar non-peak values compared with experiment results [11] and the simulation results by other researchers [5][11]. Using finer mesh and lower initial pressure for the air portion (pink curve) can generate larger peaks (when compared with the blue curve of the original mode), which are even closer to the experiment results compared with the over-estimated peak values by some of the numerical methods.

![Figure 9](image)

(a) Published results[5][11]  (b) Current simulation results

Figure 9 Pressure Results Comparison

### 3.3 3D sloshing in cylinder tank (under earthquake) by two phase flow in FEA

In the 3D simulation case, seismic performance of a 70% filled vertical steel tank is studied. Information of the tank is given in Table 1 and the ground acceleration record (1994 Northridge earthquake) is as shown in Figure 10. Techniques introduced in 3.2 are used in the simulation. Water and air are simulated with brick elements with ALE formulation; flexible steel container is simulated with shell elements. By doing so, the bi-phase flow between the fluid and the air and the fluid-structure interaction between the tank and the water can be well realized. The finite element model of the problem is given in Figure 11.

![Figure 11](image)

**Table 1: Information of the simulation problem**

<table>
<thead>
<tr>
<th>Geometric information</th>
<th>H (m)</th>
<th>R (m)</th>
<th>$t_{\text{side}}$ (m)</th>
<th>$t_{\text{bottom}}$ (m)</th>
<th>$t_{\text{top}}$ (m)</th>
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<td></td>
<td>21.96</td>
<td>7.32</td>
<td>0.0109</td>
<td>0.0109</td>
<td>0.005</td>
</tr>
<tr>
<td>Material information</td>
<td>$\rho_{\text{steel}}$ (kg/m$^3$)</td>
<td>$E_{\text{steel}}$ (N/m$^2$)</td>
<td>$\nu$</td>
<td>$\rho_{\text{water}}$ (kg/m$^3$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7850</td>
<td>2.1E11</td>
<td>0.3</td>
<td>1000</td>
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Simulated water profiles at particular time are demonstrated in Figure 12, which well reflect the physical behaviors of water sloshing under horizontal seismic excitation. Figure 13-a shows the time-history of density, indicating the free surface profile, on the demarcation line between air and water at the left most location of the fluid volume. Frequency domain analysis of Figure 13-a demonstrates sloshing frequency at 0.25Hz (Figure 13-b), a good agreement with the code prediction convective frequency by Eq (10). Figure 14-a shows the impulsive pressure time history at the bottom of the tank; Figure 14-b gives the PSD of the time series. Again impulsive frequency between 5.5 Hz and 6.2 Hz shows reasonable match with the theoretical results given in Eq (11).
For the performance of the tank, Figure 15-a shows contours for Von Mises stress at $t=6.0s$. According to the figure, the yielding has occurred at locations close to the base of the tank. Figure 15-b further demonstrates that local buckling occurs together with steel yielding. The deformation mode is consistent with the well-known elephant failure. The information, obtained based on actual FSI, is important in evaluating tank seismic performance under earthquake actions.

4. Concluding remarks

The paper conducted nonlinear sloshing simulation in both 2D and 3D water tanks. It is found that:

1. Both the CFD method through AcuSolve and the FEA method through RADIOSS used in this paper are competent in simulating 2D sloshing problems, by demonstrating reasonably matched sloshing profile, sloshing period, sloshing wave elevation and sloshing peak pressure compared with analytical solution and published experimental and numerical data.

2. The CFD simulation introduced in this paper is powerful in representing nonlinear sloshing behaviors under intensive sloshing magnitude.

3. FSI is considered in the 3D sloshing problem. The simulation rationally reflects the 3D sloshing behaviors under seismic excitation and well represents the fundamental frequencies of the convective and the impulsive components against analytical results.
4. With the consideration of FSI, detailed observation of the flow behaviors and accurate evaluation of tank seismic performance can be achieved, which provide reliable information for seismic design of LNG or storage tank design.

5. All the above developed works were incorporated in HyperWorks for pre/post processing and numerical crunching.

5. References