Outline

• Introduction to Antenna Analysis

• Computational Electromagnetics (CEM)

• CEM Solver Technologies for Antenna Modeling
  ➢ Full wave Solutions (MoM, MLFMM, FEM, FDTD)
  ➢ Asymptotic Solutions (PO, RL-GO, UTD)
  ➢ Hybrid Solutions

• Antenna Arrays
  ➢ Infinite Arrays
  ➢ Finite Arrays

• Advanced Topics
  ➢ Characteristic Mode Analysis – CMA
  ➢ Machine Learning for Antenna Design and Optimization

• Antenna Modeling and Simulation in Education and Further Reading
INTRODUCTION TO ANTENNA ANALYSIS
Electromagnetics

Maxwell's equations for electromagnetism have been called the "second great unification in physics" after the first one realized by Isaac Newton.

Maxwell’s Equations

\[ \nabla \times \mathbf{H} = J_v + \varepsilon \frac{d\mathbf{E}}{dt} \]

\[ \nabla \times \mathbf{E} = -\mathbf{M}_v - \mu \frac{d\mathbf{H}}{dt} \]

\[ \nabla \cdot \mathbf{H} = \frac{1}{\mu} \sigma_m \]

\[ \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon} \sigma_e \]

\[ \mathbf{E} = -j\omega\mu \mathbf{A} + \frac{1}{j\omega\varepsilon} \nabla (\nabla \cdot \mathbf{A}) \]

\[ \mathbf{E} = -j\omega\mu \int_V dV' G(r, r') \cdot \mathbf{J}(r') \]

\[ G(r, r') = \frac{1}{4\pi} \left[ \mathbf{I} + \frac{\nabla \nabla}{k^2} \right] G(r, r') \]

James Clerk Maxwell
(1831-1879)
Invention of Radio

Guglielmo Marconi (1874 – 1937)

- 12 December 1901, using a 500-foot antenna for reception, the message was received at Signal Hill in St John's, Newfoundland (now part of Canada) signals transmitted by a high-power station at Poldhu, Cornwall, England
- The distance between the two points was about 2,200 miles (3,500 km).

- Founded Marconi's Wireless Telegraph Company of Canada in 1903
- Later Renamed as “Canadian Marconi Company” in 1925
- Now called CMC Electronics, a wholly owned subsidiary of Esterline Corporation - http://www.cmcelectronics.ca
Invention of Radio

Jagadish Chandra Bose (1858 – 1937)

• During a November 1894 public demonstration at Town Hall of Kolkata, Bose ignited gunpowder and rang a bell at a distance using millimetre range wavelength microwaves.

• Bose wrote in a Bengali essay, Adrisya Alok (Invisible Light),

  "The invisible light can easily pass through brick walls, buildings etc. Therefore, messages can be transmitted by means of it without the mediation of wires."

On 14 September 2012, Bose's experimental work in millimetre-band radio was recognized as an IEEE Milestone in Electrical and Computer Engineering, the first such recognition of a discovery in India

Antennas Today…
Analyzing Antennas

• Based on Solving Governing Equations of underlying Physics
• Expressed in the form of Differential or integral Equations
• Solution of Governing Equations based on various Boundary Conditions of a specific problem
• Analytical Solutions are possible when the problem at hand is simple enough to apply boundary conditions

Maxwell’s Equations for Electromagnetics

<table>
<thead>
<tr>
<th>Name</th>
<th>Integral equations</th>
<th>Differential equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’s law</td>
<td>∬∬ G1 E · dS = \frac{1}{\varepsilon_0} ∬ ∬ F D V</td>
<td>\nabla · E = \frac{\rho}{\varepsilon_0}</td>
</tr>
<tr>
<td>Gauss’s law for magnetism</td>
<td>∬ ∬ G1 B · dS = 0</td>
<td>\nabla · B = 0</td>
</tr>
<tr>
<td>Maxwell–Faraday equation</td>
<td>∬ ∂G E · dS = - \frac{d}{dt} ∬ ∫ B · dS</td>
<td>\nabla × E = -\frac{\partial B}{\partial t}</td>
</tr>
<tr>
<td>(Faraday’s law of induction)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ampere’s circuit law (with</td>
<td>∬ ∂G B · dS = \mu_0 \left( ∬ ∫ J · dS + \varepsilon_0 \frac{d}{dt} ∬ ∫ E · dS \right)</td>
<td>\nabla × B = \mu_0 \left( J + \varepsilon_0 \frac{\partial E}{\partial t} \right)</td>
</tr>
<tr>
<td>Maxwell’s addition)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Maxwell’s Equations for Electromagnetics

https://en.wikipedia.org/wiki/Maxwell's_equations

James Clerk Maxwell
(1831-1879)
Analyzing Antennas

Solving Maxwell’s Equations

- Electromagnetic field behavior is governed by Maxwell’s equations
- Expressed in terms of fields (E, H) and sources (J, M)

\[
\nabla \times \mathbf{H} = \mathbf{J}_v + \varepsilon \frac{d \mathbf{E}}{dt} \\
\nabla \times \mathbf{E} = -\mathbf{M}_v - \mu \frac{d \mathbf{H}}{dt} \\
\n\nabla \cdot \mathbf{H} = \frac{1}{\mu} \sigma_m \\
\n\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon} \sigma_e
\]

Solving for Electric Field in terms of Vector Potential \( \mathbf{A} \) which is obtained using Free Space Green’s Function, \( \mathbf{G} \)

\[
\mathbf{E} = -j\omega \mu \mathbf{A} + \frac{1}{j\omega \varepsilon} \nabla (\nabla \cdot \mathbf{A})
\]

\[
\mathbf{E} = -j\omega \mu \int_V d\mathbf{r}' G(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')
\]

\[
G(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \left[ I + \frac{\nabla \nabla}{k^2} \right] G(\mathbf{r}, \mathbf{r}')
\]

\( \mathbf{A} = \) Vector Potential
\( \mathbf{G} = \) Green’s Function
Analyzing Antennas

Solving Maxwell’s Equations

• A complete description of an EM problem should include information about
  ✓ Differential/Integral equations (Maxwell’s equations)
  ✓ Boundary conditions

• Tangential components of an E field is continuous across an interface and zero on a perfectly conducting (PEC) surface

• Tangential component of an H field is discontinuous across an interface (where a surface current exists)
Antennas – Analytical Approach

Dipole Antenna

Vector Potential
\[
A(x, y, z) = \frac{\mu}{4\pi} \int I_e(x', y', z') \frac{e^{-jkR}}{R} d\ell'
\]

\[ l \ll \lambda \quad I_e \text{ is constant } I_0 \text{ (typically length is less than } \lambda/50) \]

\[ I \text{ is in the range of } \lambda/50 \text{ to } \lambda/10 \]

\[
I_e(x' = 0, y' = 0, z') = \begin{cases} 
\ddot{z} I_0 \left( 1 - \frac{2}{l} z' \right), & 0 \leq z' \leq l/2 \\
\ddot{z} I_0 \left( 1 + \frac{2}{l} z' \right), & -l/2 \leq z' \leq 0 
\end{cases}
\]

\[
A(x, y, z) = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I_e \left( 1 \pm \frac{2}{l} z' \right) \frac{e^{-jkR}}{R} dz' + \int_{l/2}^{l} I_0 \left( 1 - \frac{2}{l} z' \right) \frac{e^{-jkR}}{R} dz' 
\]

\[ l > \lambda/10 \]

\[
I_e(x' = 0, y' = 0, z') = \begin{cases} 
\ddot{z} I_0 \sin \left[ k \left( \frac{l}{2} - z' \right) \right], & 0 \leq z' \leq l/2 \\
\ddot{z} I_0 \sin \left[ k \left( \frac{l}{2} + z' \right) \right], & -l/2 \leq z' \leq 0 
\end{cases}
\]

??? Gets more complicated

Current Distribution

Directivity
Analyzing Antennas – Modeling and Simulation

Dipole Antenna
Analyzing Antennas – Modeling and Simulation
Dipole Antenna with Surrounding Environment

By itself

Infront of a Conducting Cylinder

Infront of a Conducting Sphere

Infront of a Dielectric Sphere

Electric Field through the Dielectric Sphere
COMPUTATIONAL ELECTROMAGNETICS
Computational Electromagnetics (CEM)

- CEM is the numerical solution of Maxwell’s equations
  - CEM has become an indispensable industrial tool

**Computational cost (CPU time & memory) must be as low as possible**

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{\varepsilon} \frac{d \mathbf{E}}{dt} \\
\nabla \times \mathbf{E} = -\mathbf{M} - \mu \frac{d \mathbf{H}}{dt} \\
\n\nabla \cdot \mathbf{H} = \frac{1}{\mu} \sigma_m \\
\n\n\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon} \sigma^e \\
\n\]
Computational Electromagnetics (CEM)

120MHz VHF Comm Antenna
Computational Electromagnetics (CEM)

1.8GHz LTE Antenna
Computational Electromagnetics (CEM)
Altair Antenna Simulation Solutions

Altair Feko
High Frequency EM Simulations

https://altairhyperworks.com/product/Feko

Altair WinProp
Wave Propagation & Radio Network Planning

https://altairhyperworks.com/product/Feko/WinProp-Propagation-Modeling
Antennas in Product Development

**Altair Feko** - High Frequency EM Simulations

**Altair WinProp** - Wave Propagation & Wireless Network Planning

- Antenna Design
- Antenna Placement
- Virtual Flight Test
Antennas in Product Development

Altair Feko - High Frequency EM Simulations

Altair WinProp - Wave Propagation & Wireless Network Planning

Connectivity in City Environment

Connectivity in Indoor Environment

Mobile Device

Base Station

Wi-fi Router
CEM Solver Technologies

A basic knowledge of CEM Solver Technologies is required to understand the advantages and disadvantages of each and how these affect their applicability to solve different classes of antenna problems.

- **Full Wave Solutions**
  - Method of Moments (MoM)
  - Multilevel Fast Multipole Method (MLFMM)
  - Finite Element Method (FEM)
  - Finite Difference Time Domain (FDTD)

- **Asymptotic Solutions**
  - Physical Optics (PO)
  - Large Element Physical Optics (LE-PO)
  - Ray Launching Geometrical Optics (RL-GO) (also known as Shooting and Bouncing Ray – SBR method)
  - Uniform Theory of Diffraction (UTD)

Full wave solutions solve Maxwell Equations accurately and provide reliable results provided a good CAD model and mesh is available.

Asymptotic solutions also solve Maxwell Equations, but with appropriate assumptions and approximations. They also can provide reasonably accurate results, provided the approximations and assumptions are properly considered during the simulation process.
CEM Solver Technologies

• Hybrid Solutions
  ➢ FEM/MoM/MLFMM
  ➢ MoM/PO
  ➢ MLFMM/PO
  ➢ MoM/LE-PO
  ➢ MLFMM/LE-PO
  ➢ MoM/RL-GO
  ➢ MoM/UTD

While full wave solutions are accurate, they are computationally expensive when applied to electrically large structures.

While asymptotic solutions may provide an alternative, they may not be suitable for modeling complex antenna geometries.

Hybrid solutions that combine, both full wave and asymptotic solutions can facilitate simulation of electrically large antenna problems with less computational resources, but at the same time providing required accuracy.
CEM Solver Technologies

Hybridization to solve large and complex problems

Asymptotic Solutions
(high-frequency approximation)

Full-wave Solutions
(physically rigorous solution)
Antennas

Wire Antennas – MoM and MLFMM

Planar Antennas – Planar Green’s Function, MoM, FEM, FDTD

Antenna Arrays – Planar Green’s Function, MoM, MLFMM, FEM, FDTD, Finite Array Tool
Antennas

Horns, Apertures and Lenses – MoM, MLFMM, FEM and RL-GO

Reflector Antennas – MLFMM, PO, LE-PO, RL-GO

Mobile and Wireless Antennas – MoM, FEM, FDTD
FULL WAVE SOLUTIONS
# Full Wave Solutions

<table>
<thead>
<tr>
<th></th>
<th><strong>Field method</strong></th>
<th><strong>Source method</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Differential form of Maxwell’s Equations)</td>
<td>(Integral form of Maxwell’s Equations)</td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td>Electromagnetic fields</td>
<td>Currents and charges</td>
</tr>
<tr>
<td><strong>Equations</strong></td>
<td>Differential equations</td>
<td>Integral equations</td>
</tr>
<tr>
<td><strong>Discretization</strong></td>
<td>Volumetric (Tetrahedral, Voxels etc)</td>
<td>Surface (segments and triangles)</td>
</tr>
<tr>
<td><strong>Radiation Boundary</strong></td>
<td>Special Absorbing Boundary Conditions (ABCs) must be introduced (&quot;Air Box&quot; around the radiating structure.)</td>
<td>Exact treatment by using free space Green’s Function (No “Air Box” Needed)</td>
</tr>
</tbody>
</table>
| **Solutions**        | Finite Element methods (FEM)  
Finite Difference Time Domain (FDTD) | Method of Moments (MoM)  
Adaptive Cross Approximation (ACA)  
Multilevel Fast Multipole Method (MLFMM) |
METHOD OF MOMENTS
Method of Moments (MoM)

- Create CAD Model of the geometry
- Create surface mesh – triangles
- Applying the equivalence principle electric or magnetic currents assumed to be unknowns
- RWG basis functions are used
- A set of linear equations are formed

\[ Z I = V \]

\( Z = \) NXN complex matrix
\( I = \) Unknown current vector
\( V = \) Known Excitation vector

- Solving this equation, unknown currents on each triangle is found
Method of Moments (MoM)

Antenna Characteristics can be found from the currents calculated:
• Near- or Far-fields
• Input impedances
• S-parameters etc
MoM Examples - Wire Discone Antenna

- Lowest -10 dB S11 frequency = 50 MHz

Feeding with voltage source at wire port

S11 vs. frequency

VoltageSource1 [S11]
MoM Examples – Broadband Helix

Frequency Band: 800MHz to 1.2 GHz
MoM Examples - Printed Log Periodic Antenna

- **Finite substrate:**
  - FR4
  - $h = 1.5$ mm
  - $\varepsilon_r = 4.35$
  - 18 elements

- **Design:**
  - Center frequency = 1 GHz
  - Bandwidth = 70%
  - Gain = 9 dBi

MoM Examples - Printed Log Periodic Antenna

- **Finite substrate:**
  - FR4
  - $h = 1.5$ mm
  - $\varepsilon_r = 4.35$
  - 18 elements

- **Design:**
  - Center frequency = 1 GHz
  - Bandwidth = 70%
  - Gain = 9 dBi
MoM Examples – CPW fed Bowtie Antenna

- **Finite substrate:**
  - Rogers – RT/duroid - 5880
  - h = 1.575 mm
  - ε_r = 2.2

- **Design:**
  - Center frequency = 9 GHz
  - 50 Ω
  - Expect wide band match

\[
\text{VSWR}
\]

**MoM mesh**
MoM Examples – Microstrip Patch Antenna

- **Finite substrate:**
  - $h = 7.5$ mm
  - $\varepsilon_r = 2.0$
  - Ground Plane = 277 mm

- **Design:**
  - Frequency = 1.62 GHz
  - $50 \, \Omega$
MoM Examples – Microstrip Patch Antenna Array

Array:
- 106 elements
- Size of the Panel = 1.8796m x 0.8636m x 0.048m
- Frequency = 1.62 GHz

Triangles: 47,148  
Number of Unknowns = 131,565  
Z Matrix Size = 131,565 x 131,565 (Complex numbers)

MoM Memory Requirement = 125 GBs

Limitation of Computer Hardware used:
Dell Precision 5720  
3.70GHz quad core 64-bit processor with 4 cores  
Memory of 64GBs  
Microsoft Windows 10 Operating System.  
(< US $3,000)
MULTILEVEL FAST MULTIPOLE METHOD
MLFMM
Computational Complexity of MoM

MoM based on the solution of a system of linear equations

\[ Z I = V \quad \quad I = Z^{-1} V \]

Impedance matrix \( Z \) describes interaction of \( n.\)th element with \( m.\)th element

\[ Z = \begin{bmatrix}
* & * & * & * & * \\
* & * & * & z_{mn} & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\end{bmatrix} \]

source element \( n \)

observation element \( m \)

\[ z_{mn} \]

\[ \rightarrow \text{LU-decomposition requires } O(N^3) \text{ operations and } O(N^2) \text{ memory} \]
Resource Requirement

Example:
Automotive simulation
at 2 GHz instead of 1 GHz:

\[ f \rightarrow 2f \]
\[ N \rightarrow 4N \]

<table>
<thead>
<tr>
<th>Complexity</th>
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<tbody>
<tr>
<td>( O (N^2) )</td>
<td>64</td>
</tr>
<tr>
<td>( O (N^2) )</td>
<td>16</td>
</tr>
<tr>
<td>( O (N) )</td>
<td>4</td>
</tr>
<tr>
<td>( O (N \log N) )</td>
<td>(4 \cdot \left(1 + \frac{\log 4}{\log N}\right) &lt; 5)</td>
</tr>
<tr>
<td>( O (N \log^2 N) )</td>
<td>(4 \cdot \left(1 + \frac{2 \log 4}{\log N} + \frac{\log^2 4}{\log^2 N}\right) &lt; 6)</td>
</tr>
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</table>
Multilevel Fast Multipole Method (MLFMM)

• **Multilevel implementation:**
  - Divide space into boxes
  - Aggregation (A)
  - Translation (T)
  - Disaggregation (D)
Resource Requirement

Example:
Automotive simulation
at 2 GHz instead of 1 GHz:
\[ f \rightarrow 2f \]
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MLFMM – Microstrip Patch Antenna Array

Array:
• 106 elements
• Size of the Panel = 1.8796m x 0.8636m x 0.048m
• Frequency = 1.62 GHz

Surface Currents and Radiation Pattern

Triangles: 47,148
Number of Unknowns = 131,565
Z Matrix Size = 131,565 x 131,565 (Complex numbers)
MoM Memory Requirement = 125 GBs
MLFMM Memory Requirement = 11 GBs !!
10 mins !!
MLFMM – Microstrip Patch on A Satellite

Triangles: 216,909

Number of Unknowns = 376,722
Z Matrix Size = 376,722 x 376,722 (Complex numbers)
MoM Memory Requirement ~ 1 TBs

MLFMM Memory Requirement = 11 GBs !! In 47 mins !!
MLFMM - Analysis of a Reflector Antenna

• **Problem description:**
  • Offset parabolic reflector
  • Cylindrical horn
  • Support structure
  • 12.5 GHz
  • **18 inch aperture (19 λ)**

• **Solution:**
  • 94,000 unknowns
  • Solved with MLFMM
  • 1.2 GByte RAM
  • MoM solution would require 134 GByte RAM
FINITE ELEMENT METHOD
FEM, FEM/MOM AND FEM/MLFMM
What is the FEM?

- **FEM** = Finite Element Method
- **Full-wave method**
- Solves the differential form of Maxwell's equations:
  - Volume discretization
  - Generates Sparse matrix
- **Advantages:**
  - Well-suited to highly inhomogeneous regions
  - 3D Anisotropic Materials
  - Memory efficient

- **Disadvantages of using only FEM:**
  - Numerical dispersion
  - Entire problem space discretized
  - Requires a solution to a large set of linear equations
  - No intrinsic radiation boundary condition as with MoM
  - Requires artificial Absorbing Boundaries (normally referred to as “Air Box” for radiation problems)
What is Hybrid FEM-MoM?

- **MoM** is ideal for radiation and coupling analysis

- **MoM/SEP** → not optimal for HIGHLY inhomogeneous, geometrically complex dielectric bodies

- **FEM-MoM** uses surface integral equation as radiation boundary condition to FEM (tangential field continuity)

- Not unnecessary to discretize of 3D **free-space** (“white space”)

- **Best of both worlds:**
  - Use FEM for efficient modelling of inhomogeneous dielectrics
  - Use MoM for efficient, accurate modelling of complex wires, metallic surfaces, sources of radiation and open spaces

- **FEM-MLFMM** is same as **FEM-MoM** → more efficient for electrically large MoM

Example: base station radiation hazard analysis

\[
\hat{n} \times \vec{E}^{\text{FEM}} \bigg|_{\partial\Omega} = \hat{n} \times \vec{E}^{\text{MoM}} \bigg|_{\partial\Omega}
\]

\[
\vec{J}^{\text{FEM}} = -\vec{J}^{\text{MoM}}
\]
FEM Example – Microstrip Patch Antenna

Design:

\[ \varepsilon_r = 2.2 \]

Patch = 31.18 x 46.64
Substrate = 50 x 80 x 2.87
Frequency \( \sim \) 2.8 to 3.1 GHz
Hybrid FEM-SEP-MoM (MLFMM)

- Strengths of the Hybrid FEM-SEP-MoM (MLFMM)
  - Well suited to models with inhomogeneous dielectrics and large homogeneous dielectrics
  - Combines strengths of FEM and SEP and MoM/MLFMM solutions!
FINITE DIFFERENCE TIME DOMAIN METHOD
FDTD
Finite Difference Time Domain - FDTD

- Solves the differential form of Maxwell's equations
- Volume discretization
- Traditional advantages
  - Inhomogeneous materials easily accommodated.
  - 3D Anisotropic Materials
  - Memory efficient
  - GPU Friendly
- Traditional disadvantages
  - Boundary condition application as a stair step.
  - Numerical dispersion of propagating waves
  - Entire problem space discretised.
  - Structured mesh.
  - Run-time dependent on time taken for energy to propagate out of problem space.

FDTD - Voxel Mesh Generator

- Discretization of models into voxel elements for FDTD solution
- Supports non-uniform meshing of geometry and mesh parts
- Aligned to key points and boundaries
- Mesh settings
  - Standard / fine / coarse auto setting
- Observes simulation frequency and material properties
  - Custom setting
  - Advanced settings
- Growth rate
- Aspect ratio
- Handling of small geometry features
FDTD Example – Microstrip Patch Antenna

Design:
\[ \varepsilon_r = 2.2 \]
Patch = 31.18 x 46.64
Substrate = 50 x 80 x 2.87
Frequency ~ 2.8 to 3.1 GHz
ASYMPTOTIC AND HYBRID SOLUTIONS
Asymptotic Methods Motivation and Application

• **Motivation:**
  - To solve extremely large models
  - Larger than MoM or MLFMM can solve
  - with available computational resources

• **Conditions of applicability:**
  - Radiator/source is localized
  - Radiator/source is far away
  - Structure features are large in terms of wavelength
  - Typically used for antenna placement or scattering analysis
HYBRID MOM/PO
Physical Optics (PO)

\[ \mathbf{J}(\mathbf{r}) = \hat{n} \times \mathbf{H}(\mathbf{r}) = \hat{n} \times \left[ \mathbf{H}^i(\mathbf{r}) + \mathbf{H}^r(\mathbf{r}) \right] \]

\[ \mathbf{J}(\mathbf{r}) = 2\hat{n} \times \mathbf{H}^i(\mathbf{r}) \]
Hybrid MoM/Physical Optics (PO) Technique

Decomposition of domain into MoM and asymptotic region

Two types of coupling:

- $J^{MoM}$ radiates H causing asymptotic currents
- $J^{asym}$ radiates E which must be considered in the MoM integral equation

$$\vec{E} \left\{ J^{MoM} \right\}_{tan} + \vec{E} \left\{ J^{asym} \right\}_{tan} = -\vec{E}_{i, tan}$$
Discretization of Currents in MoM/PO

- PO currents represented exactly like MoM currents
- Triangular mesh
- Same basis functions

\[ \bar{J}^{PO} = \sum_{n=1}^{N} \alpha_n \cdot \bar{f}_n \]

- Meshing guidelines
  - Same as for metallic MoM

**PO Unknowns** $N$: MoM Unknowns $M$

**Storage Requirement**
- $MXM$ – MoM Matrix
- $NXM$ – MoM/PO Coupling Matrix

Continuous current flow on the boundary between MoM and PO regions
Hybrid MoM/PO Example – Offset Reflector

Frequency: 6.25 GHz
Offset reflector diameter ≈ 10.4λ
Focal distance: ≈ 5.7 λ

Phi = 0 deg

Phi = 90 deg
Hybrid MoM/PO Example - Monocone on Ship at 500MHz

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>500MHz</td>
<td>200 λ</td>
<td>23.3 λ</td>
<td>61.7 λ</td>
</tr>
</tbody>
</table>

**MoM-PO Hybrid - Coupled**

- Number of Triangles: 3,513,348
- Memory Required: 33 GBs
- Time: 1.1 hours

MoM for Monocone

PO for the rest of the model
Large Element PO Formulation (LE-PO)

Electrically large triangular patches for PO:

- Traditional RWG (Rao-Wilton-Glisson) basis functions $f_n$ require electrically small mesh elements ($\lambda/6 \ldots \lambda/12$):

  $$f_n(r) = \begin{cases} \frac{l_n}{2A_n} \bar{p}_n^+, & r \in T_n^+ \\ \frac{l_n}{2A_n} \bar{p}_n^-, & r \in T_n^- \\ 0 & \text{otherwise,} \end{cases}$$

- Incorporation of linear phase term into basis function allows the use much larger mesh elements (several $\lambda$):

  $$f_{nh}(r) = \begin{cases} \frac{l_n}{2A_n} \bar{p}_n^+ \cdot e^{-jk_n \cdot (\bar{p}_n^+ - \bar{p}_n^-)}, & r \in T_n^+ \\ \frac{l_n}{2A_n} \bar{p}_n^- \cdot e^{-jk_n \cdot (\bar{p}_n^- - \bar{p}_n^-)}, & r \in T_n^- \\ 0 & \text{otherwise,} \end{cases}$$
Hybrid MoM/PO Example – Offset Reflector

Frequency: 6.25 GHz
Offset reflector diameter $\approx 10.4\lambda$
Focal distance: $\approx 5.7\lambda$

Phi = 0 deg
Phi = 90 deg
### Monocone on Ship at 500MHz

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>500MHz</td>
<td>200 $\lambda$</td>
<td>23.3 $\lambda$</td>
<td>61.7 $\lambda$</td>
</tr>
</tbody>
</table>

- **LE-PO for the rest of the model**
- **MoM for Monocone**

### MoM-LE-PO Hybrid - Coupled
- Number of Triangles: 10,974
- Memory Required: **238MBs (33GBs for PO)**
- Time: **2.7 mins (1.1 hours for PO)**
### Monocone on Ship at 500MHz

<table>
<thead>
<tr>
<th>Frequency</th>
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<td>500MHz</td>
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</tr>
</tbody>
</table>

### MoM-LE-PO Hybrid - Coupled

- Number of Triangles: 10,974
- Memory Required: **238MBs**
- Time: **2.7 mins**

![Phi = 0 degs](image1)

![Phi = 90 degs](image2)
Iterative Hybrid MoM/PO or MLFMM/PO

1) Solve MoM/MLFMM problem ignoring PO region
   Compute $J_{PO}$ from the scattered magnetic field
   caused by $J_{MoM}$

2) The scattered electric field caused by $J_{PO}$ then radiates back into the MoM region and modifies the excitation vector

3) With the new excitation vector, repeat from 2) until $J_{MoM}$ has reached convergence
Cassegrain Reflector Antenna

MLFMM – Feed Horn + Sub reflector
LE-PO - Reflector

Full MLFMM: 0.786 hours, 12.4 GByte
Hybrid MLFMM/LE-PO: 0.129 hours, 0.43 GByte

Intel(R) Core(TM) i7-3770K CPU @ 3.50GHz; 4 parallel processes
HYBRID MOM/RL-GO
Ray-Launching Geometrical Optics (RL-GO)

- Aimed at solution of electrically very large (\( > 20 \lambda \)) structures
- E.g. Lenses, reflectors, large scatterers
- GO = Geometrical Optics
- Ray-launching, optical – Also known as Shooting and Bouncing Ray (SBR) Method
- Interaction with MoM structures via ray-launching principles

**Advantages:**
- Explore RL-GO when PO has failed
- Mesh can be very coarse (as opposed to PO) – no mesh storage problem
- Good for smooth, large structures

**Disadvantages:**
- Grazing incidence means that RL-GO sources will be sparsely placed, forcing very fine launching increment
- Reduced accuracy with many multiple reflections
MoM / RL-GO formulation

- From each source ray tubes are launched at incremental spacing, covering all directions
- Where a ray tube hits a surface, J-sources (and/or M-sources for dielectrics) are placed on the surface, based on plane wave approximation
- As a ray tube bounces between surfaces, a source(s) is added at every interaction point
- Total solution field = incident fields + all RL-GO sources (reflected fields)
RL-GO Example: Reflector Antenna

• Parabolic reflector
• Circular horn antenna feed
• Fundamental waveguide mode excitation
• 8 GHz
• Reflector aperture = 36 λ
RL-GO Example: Reflector Antenna

Near Fields

- **RL-GO with Eq Source**
  - Memory: 62 MBs
  - CPU Time: 4 mins

- **MLFMM with Eq Source**
  - Memory: 4 GBs
  - CPU Time: 3 mins

- **MLFMM with Horn Feed**
  - Memory: 4.4 GBs
  - CPU Time: 7 mins
**RL-GO Example: Lens Antenna**

**Dielectric Lens Antenna**

Frequency: 30GHz \( \varepsilon_r = 6 \)
Diameter = 10cm \( \text{Tan} \delta = 0.005 \)

\[ E_x = \cos^4(\theta) \text{ where } 0 \leq \theta \leq \frac{\pi}{2} \]

**MLFMM**

- Memory: 15.3GBs
- CPU Time: 8mins

**RL-GO**

- Memory: 97 MBs
- CPU Time: 11 secs
HYBRID MOM/UTD
Uniform Theory of Diffraction (UTD) - Motivation

- PO and RL-GO can be computationally expensive when:
  - Problem extremely large in wavelengths (>1,000s of wavelengths)
  - Diffraction is important
  - Multiple interactions involving reflections and diffractions are important
- For such problems UTD may be suitable
- UTD is based on field ray tracing using reflection, diffraction, and creeping wave calculations
- Computational Complexity remains constant if the problem is suitable for UTD

<table>
<thead>
<tr>
<th>method</th>
<th>formulation</th>
<th>CPU–time</th>
<th>memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>MoM</td>
<td>current–based</td>
<td>$f^4...6$</td>
<td>$f^4$</td>
</tr>
<tr>
<td>PO</td>
<td>current–based</td>
<td>$f^2$</td>
<td>$f^0$</td>
</tr>
<tr>
<td>UTD</td>
<td>ray–based</td>
<td>$f^0$</td>
<td>$f^0$</td>
</tr>
</tbody>
</table>
Uniform Theory of Diffraction (UTD)

**Geometry restrictions:**
- PEC or lossy metal structures
- Also PEC with coatings/thin dielectric sheet
- Must consist of flat polygonal plates
- Single cylinder allowed
- Edge length/diameter of plates must be $> 1\lambda$
- “Mesh” is the same as the plates (i.e. CAD)

**Types of rays considered:**
- Direct rays
- Reflected rays (also multiple edge)
- Edge diffracted rays
- Corner diffracted rays
- Combinations/multiples of reflections and diffractions
- Creeping rays
Hybrid MoM/UTD - Satellite Structure

- Geometry consists of rectangular plates
- Box structure with two reflector panels
- Well suited for MoM-UTD Hybrid Method
- Excitation: single 1/4 λ monopole (MoM)

6GHz  
1 m = 20 λ
Hybrid MoM/UTD - Satellite Structure

15GHz

1 m = 50 λ

Method | 6 GHz | 15 GHz
--- | --- | ---
MLFMM | 14.1 GByte | ---
UTD | 1.0 MByte | 1.0 MByte
UTD Example - Radar on a ship Deck

Representative Radar Antenna @ 10GHz

- Import as pattern point source to location 50 cm above deck
- Specify far-field calculation in front of antenna (-60°, +60°)

<table>
<thead>
<tr>
<th>Distance</th>
<th>Radiation</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>120m</td>
<td>4,000 λ</td>
<td>10GHz</td>
</tr>
<tr>
<td>14m</td>
<td>467 λ</td>
<td></td>
</tr>
<tr>
<td>37m</td>
<td>1,233 λ</td>
<td></td>
</tr>
</tbody>
</table>
RADIATION PATTERN VS. AZIMUTH SCAN ANGLE

- At 340° worst tower blockage evident (peak pattern gain shown in parenthesis)

Radiation pattern as a function of scan angle

Computational cost (laptop, single process):
2 Mbytes
6 min/pattern
UTD - Detailed Analysis of Pattern at 340°

- Recalculate at 340° - much finer far-field sampling
- Lobes due to path gain now all resolved
  - Antenna is 16.67λ above deck
- Computational cost (laptop, single process): 2 Mbytes, 6 hours

10GHz
ANTENNA ARRAYS
Arrays Using Periodic Boundary Conditions (PBCs)

FEM or MoM can be used for infinite Periodic structures using PBCs

Helix Antenna Array Simulation

- Radiation pattern analysis for arbitrarily large arrays (1D or 2D rectangular arrays)
- Simulate single element of array, integrate for antenna pattern using array factor

21 x 21 Array

Solution Method | Memory
--- | ---
PBC | 0.5 MByte
MLFMM | 4.5 GBs
FINITE ANTENNA ARRAYS
DOMAIN GREENS FUNCTION METHOD - DGFM
DGFM – Efficient Method for Finite Antenna Arrays

- **Analysis Based on Solving One Array Element at a Time**
  - Accounts for *Edge Effects* of Finite Arrays
  - **Mutual Coupling** is Accounted for When Calculating Self-Interaction Matrix of the Element by Using a Modified Green’s Function
  - **The Computational Complexity** Scales Much Better – By Solving Smaller Matrix Equations

$$
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{I}}_{DGFM}
\end{bmatrix}
= 
\begin{bmatrix}
\dot{\mathbf{1}}_{11} & \dot{\mathbf{1}}_{12} & \dot{\mathbf{1}}_{13} \\
\dot{\mathbf{1}}_{21} & \dot{\mathbf{1}}_{22} & \dot{\mathbf{1}}_{23} \\
\dot{\mathbf{1}}_{31} & \dot{\mathbf{1}}_{32} & \dot{\mathbf{1}}_{33}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
$$

DGFM – Efficient Method for Finite Antenna Arrays

- Linear array
- Circular or cylindrical
- Custom array

<table>
<thead>
<tr>
<th></th>
<th>MoM</th>
<th>DGFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU time</td>
<td>1.5 hours</td>
<td>28.7 min</td>
</tr>
<tr>
<td>Total Memory Usage</td>
<td>7.54 GByte</td>
<td>319.4 MByte</td>
</tr>
</tbody>
</table>
ADVANCED TOPICS
CMA – Characteristic Mode Analysis

• CMA gives you fundamental physical insights that a driven simulation doesn’t give you.
• CMA can help in antenna design: how to modify the shape, where to place excitations and loads.

“Putting Physics Back into Simulations”

Machine Learning for Antenna Design and Optimization

“Machine learning is a field of study that gives computers the ability to learn without being explicitly programmed.” - Arthur Samuel, Computer Scientist 1959

Alternative Description:
Machine learning is a family of algorithms that help make predictions from data sets

Simulation data with multiple variables
Supervised Data needed for ML

Machine Learning with Regression
Build a mathematical model that defines the goal (Return Loss of the Antenna etc.) as function of geometry variables

Optimize using ML Model
Machine Learning for Antenna Design and Optimization

Antenna Design and Optimization Using Machine Learning

On-Demand Short Course

Machine learning is a method of data analysis that automates analytical model building. As the complexity of antennas increases each day, antenna designers can take advantage of machine learning to generate trained models for their physical antenna designs and perform fast and intelligent optimization on these trained models. Using the trained models, different optimization algorithms and goals can be run quickly, in seconds, that can be utilized for comparison studies, stochastic analysis for tolerance studies etc.

This short course presents the process of fast and intelligent optimization by adopting the Design of Experiments (DOE) and Machine Learning using Altair FEKO. We discuss specific examples that showcase the advantages of using DOE for antenna design and optimization.

Access Short Course

Speakers

Dr. C.J. Reddy
Vice President, Business Development - Electromagnetics

Dr. Reddy was awarded the U.S. National Research Council (NRC) Resident Research Associateship at NASA Langley Research Center. He is currently a Fellow of IEEE, AGES and AIAA and has published 37 journal papers, 77 conference papers and 18 NASA Technical Reports to date.

Gopinath Gampala
Technical Regional Manager

Gopi graduated from University of Mississippi with a Master’s degree in computational electromagnetics in 2007 and working in the field of CAD since then. He is a member of IEEE and published extensively on topics like High-impedance surfaces, Low-profile antennas, LTE, Radar, Characteristic Mode Analysis, 5G and Machine Learning.

https://web.altair.com/antenna-design-optimization-machine-learning-on-demand
ANTENNA MODELING AND SIMULATION IN EDUCATION
Fusing Theory and Simulations in Lab

University of Michigan – Dearborn

- Simulate the “experiment” with Altair Feko
- Correlate simulation results with theory
- Experiment in the lab
- Correlate measured lab data with theory and simulations

Altair Feko Student Edition

**Free** for Students and Faculty to Download at

Feko – Comprehensive Electromagnetic Simulations

WinProp – Wave Propagation and Radio Network Planning Tool


altairuniversity@altair.com
Learn Electromagnetic Simulation with Altair Feko

Altair Feko is an environment to solve electromagnetic problems. This book takes the reader through the basics of broad spectrum of EM problems, including antennas, the placement of antennas on electrically large structures, microstrip circuits, RF components, the calculation of scattering as well as the investigation of electromagnetic compatibility (EMC). The concepts are explained with examples and step-by-step tutorials after each section. Moreover, the users will also be guided with videos to make the learning experience fast and effective.

DOWNLOAD the Free eBook

1. Introduction
2. Wire Dipole and Monopole Antennas
3. Wire Loop Antennas
4. Microstrip Patch Antennas
5. Microstrip Based Filters and Feed Networks
6. Broadband Dipole Antennas
7. Traveling Wave and Broadband Antennas
8. Frequency Independent Antennas
9. Horn Antennas
10. Reflector Antenna
Further Reading on Modeling and Simulation Methods

**Computational Electromagnetics for RF and Microwave Engineering**
2nd Edition
David B. Davidson
Cambridge University Press

**Handbook of Reflector Antennas and Feed Systems Volume II**
Feed Systems
Sudhakar Rao, Lotfollah Shafai, Satish K. Sharma
Artech House

**CHAPTER 2**
Numerical Methods

Kubilay Sertel, Ohio State University
C. J. Reddy, EM Software & Systems (EMSS) USA Inc.
Questions

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cjreddy@altair.com

THANK YOU

altair.com

#ONLYFORWARD